Time-varying Impact of Investor Sentiment*

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ABSTRACT

I present a dynamic equilibrium model of investor sentiment in which investors form beliefs by overly extrapolating past returns. The key contribution of my model is to connect mispricing with investor sentiment through the market impact of extrapolators, and to provide novel insights into the predictability of market returns. When their wealth level is high, extrapolators drive the asset prices. In this case, their high investor sentiment makes the current asset price overvalued, and the future asset price will decline because high investor sentiment will cool down over time. Therefore, investor sentiment negatively predicts future market returns. When extrapolators' wealth level is low, high investor sentiment predicts high future returns since the market is under a price correction. I find strong support for my model in the data. My model also matches investor sentiment in surveys, and it captures many documented patterns of boom-bust cycles in the stock market.

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I. Introduction

Whether investor sentiment influences the market has been a long-standing question in the finance literature. From the Great Crash in 1929 to the Internet bubble, from the Nifty Fifty bubble to the 2008 financial crisis, each of these episodes is associated with dramatic changes in asset prices. Traditional finance theories—models in which investors have fully correct beliefs about the asset dynamics and therefore always force the asset prices to the rational present value of expected future cash flows—leave no room for investor sentiment and have considerable difficulty fitting these patterns. However, investor sentiment, which reflects excessive optimism and pessimism in investor beliefs, seems to play a central role in these phenomena. The large gap between the traditional models and the salient market episodes with dramatic asset price movements has made researchers realize that *belief-based* investor sentiment plays an important role in asset pricing dynamics.

Relying on survey evidence, recent studies have highlighted the concept of extrapolation—making forecasts about future returns based on past realized returns—in understanding the dynamics of investor beliefs. Extrapolation implies that investors tend to believe that asset prices continue to increase after a sequence of high returns and fall after a sequence of low returns (Greenwood and Shleifer (2014)), and has been used to account for the excessive optimism and pessimism in the market (Barberis, Greenwood, Jin, and Shleifer (2016) and Barberis (2011)).[1] As a result, in this paper, I use extrapolation to characterize belief-based investor sentiment. But despite its prevalence in surveys and its importance in investors' portfolio choice, there is no strong empirical evidence that belief-based investor sentiment influences the aggregate stock market. In other words, extrapolation alone leaves the impact of investor sentiment unsolved: for instance, in Greenwood and Shleifer (2014), investor beliefs reported in the survey have only insignificant predictive power over future aggregate stock market returns.

In this paper, I provide a parsimonious approach to reveal the impact of belief-based investor sentiment. By incorporating the time-varying market impact of extrapolation, which is a notion largely overlooked by the literature, this paper documents and highlights the impact of belief-based investor sentiment on the aggregate market. Specifically, in addition to the basic extrapolation

[1] There might be other forms of extrapolation. I use extrapolation to refer to return extrapolation because according to previous studies, because it better addresses the survey expectation series. There are other studies on extrapolation behavior of investors, for example, Vissing-Jorgensen (2004), Amromin and Sharpe (2013), Koijen, Schmeling, and Vrugt (2015) and Kuchler and Zafar (2016)

framework, I incorporate the wealth dynamics of investors who extrapolate to proxy for the market impact of extrapolation. In this setting, extrapolation and its market impact together drive the asset price dynamics: when the market impact of extrapolation is high, extrapolation induces irrational demand for the risky asset and, therefore, it leads to mispricing; when the market impact is low, extrapolation simply reflects the recent market dynamics. The interaction between extrapolation and its market impact sheds new light on asset mispricings. More importantly, although investor beliefs alone do not significantly predict future aggregate market returns, they have salient predictive power after conditioning on their market impact—a result that supports both my model implications and my empirical exercise. This conditional predictive power of investor sentiment not only provides direct evidence of the impact of investor beliefs on the aggregate stock market, it also helps to reveal the underlying mechanism of the predictability of returns.

To formalize these arguments, I first develop a continuous-time dynamic equilibrium model that features two types of investors: extrapolators and fundamental investors. On the basis of past price changes, extrapolators form *investor sentiment*, or, equivalently, their perceived expectation about future risky asset returns, and they make an investment choice between a risky and risk-free asset. After consecutive positive price changes in the past, extrapolators become optimistic about future returns, and after a streak of negative price changes, they become pessimistic about future returns. However, the perceived expectations are different from the true ones and, therefore, high or low investor sentiment, in general, cannot continue for long since extrapolators will easily become disappointed by shocks in the future. Consequently, investor sentiment reverts to its mean. Moreover, the major departure from previous extrapolation models is to incorporate the wealth level of extrapolators: all other things being equal, extrapolators have a larger impact on the equilibrium asset prices when their wealth level is higher.

As in earlier models, extrapolators are met in the market by fundamental investors who serve as the counteracting forces and arbitrage against mispricing. When the risky asset is overvalued, fundamental investors short the risky asset and, therefore, correct prices downwards. If the risky asset is undervalued, fundamental investors lean against the wind and push prices upwards. However, their ability to correct mispricing depends on the wealth level of extrapolators: fundamental investors can easily correct mispricing when sentiment-driven wealth is low, while the correction takes longer when the wealth level of extrapolators is high.

This model setting generates the key result: when their wealth level is high, extrapolators drive the asset prices. In this case, high investor sentiment makes the current asset price overvalued, and the future asset price will decline because high investor sentiment will cool down over time. Therefore, investor sentiment negatively predicts future market returns. Conversely, when the wealth level is low, high investor sentiment predicts high future returns because the market is under a price correction. This predictive power of investor sentiment provides direct theoretical support for my conclusion that investor beliefs impact the aggregate stock market.

Moreover, this predictive power supports belief-based explanations of the predictability of returns—that prices temporarily deviate from the level warranted by fundamentals because of the existence of extrapolators, but they revert back as a mispricing correction gradually takes place in the future. Cassella and Gulen (2015) provide empirical support for this explanation. They define the degree of extrapolation (DOX) as the relative weight extrapolators place on recent-versus-distant past returns when they form subjective expectations for future asset returns. When DOX is high and, therefore, when investor beliefs are transitory, mispricings are corrected more quickly and price-scaled variables (such as price-to-dividend ratio) have stronger predictive power. One possible determinant of the variations in DOX is the time-varying consensus level of extrapolation among market participants. In my model, the time-varying wealth level of extrapolators effectively drives the consensus level of extrapolation in the market.

My model matches other salient patterns in the asset pricing literature. For instance, the mean-reversion of investor sentiment naturally generates a negative equity premium when investor sentiment is high, a pattern that is consistent with findings documented by Baron and Xiong (2017). The fact that my model generates a negative correlation between investors' expectations and the subsequent realized returns is consistent with the findings of Greenwood and Shleifer (2014). Moreover, the resulting countercyclical Sharpe ratio is consistent with empirical evidence documented by Lettau and Ludvigson (2010). All these asset pricing patterns seem puzzling under a rational expectations framework.

To empirically test the impact of investor sentiment on the market, I use both the CRSP value-weighted index and Gallup survey data. During the period of December 1996 to September 2011, Gallup asked individual investors to report their expectations of aggregate stock market returns in the next twelve months. Using these responses I measure investors' perceived expectations directly.

As a robustness check, I also follow Barberis, Greenwood, Jin, and Shleifer (2015) and construct an investor sentiment index, Psentiment, that is purely based on extrapolation.[2] Moreover, Gallup survey evidence helps to identify extrapolator groups, which allows me to select a reasonable proxy for the wealth level of extrapolators.

To find reasonable measurements for extrapolators, I focus on the “Households and Non-Profitable Organizations” (HNPO hereafter) sector reported in the “Financial Accounts of the United States”. Investors in this sector generally are individual investors who are less sophisticated than institutional investors. Moreover, Yang and Zhang (2017) document, first, that investor sentiment in the survey effectively drives the portfolio position of investors in stocks in the HNPO sector and, second, that such sentiment-driven investment negatively predicts returns in the following quarters.[3] Their analysis effectively indicates that, investor beliefs in the HNPO sector are associated with extrapolation. My model therefore uses the total financial assets of the HNPO sector as the proxy for the wealth level of extrapolators.

My empirical tests support the main predictions of my model. Specifically, I construct an interaction term between the Gallup survey expectations and wealth dividend ratios in the HNPO sector, and I use it to predict future market returns. I find a statistically significant and negative coefficient for the interaction term, which indicates that investor sentiment connects closely to market mispricing when the wealth level of extrapolators is high. This result holds over different predictive horizons, ranging from one to six quarters.

Furthermore, following Aiken, West, and Reno (1991), I empirically present the predictive pattern of investor sentiment conditional on different wealth levels of extrapolators.When the wealth level of extrapolator is two standard deviation above its mean, one standard deviation increase in investor sentiment measured by the Gallup survey is followed by a significant decrease of 16.2% in future twelve-months returns. This is consistent with my model implication: when extrapolators drive the market, investor sentiment reverts to its mean and leads to a negative predictive sign. This pattern remains valid when Psentiment replaces Gallup. Conversely, when extrapolators' wealth level is two standard deviation below its mean, one standard deviation increase in investor

[2] In my empirical analysis, I use both survey evidence and Psentiment to test my model predictions. If investor beliefs in the survey are mainly driven by extrapolation, then two pieces of parallel evidence should yield similar results for most of my tests. This point is strongly confirmed in my exercise.

[3] Yang and Zhang (2017) use surveys by Gallup, Inc. and the American Association of Individual Investors.

sentiment predicts a striking future market return of 35.9% : as my model implies, when sentiment-driven wealth is low, investor sentiment reflects the market valuation correction and, therefore, it positively predicts future market returns.

Implications for the Literature. This paper belongs to the burgeoning return extrapolation literature. Many works in this field try to understand the role that return extrapolation plays in the aggregate stock market. (Cutler, Poterba, and Summers (1990), De Long, Shleifer, Summers, and Waldmann (1990), Barberis et al. (2015) and Jin and Sui (2017)). Barberis et al. (2015) use return extrapolation to construct an asset pricing model that can explain central asset pricing facts such as the excess volatility puzzle, the predictability of returns, and the investor belief survey evidence in the data. Jin and Sui (2017) construct a quantitative benchmark of belief-based asset pricing models that can simultaneously explain the equity premium puzzle, the excess volatility puzzle, the predictability of returns, the low correlations between consumption and returns, and investor belief evidence in the surveys. However, all existing studies of return extrapolation ignore the role that the wealth level of extrapolators plays in asset dynamics. By incorporating the wealth dynamics of extrapolators and the counteracting forces from fundamental investors, I document a novel predictive pattern for future stock market returns.

This paper also relates to the idea of limits to arbitrage, which is one of the foundations for the behavioral finance literature. In its pioneer work, Shleifer and Vishny (1997) argue that asset mispricing may exist for a long time because arbitrage activities are limited. This is especially true when investment is delegated to portfolio managers with short investment horizons and when the arbitrage activities face noise trader risk and other risks. Abreu and Brunnermeier (2002) provide an additional argument for limited arbitrage.[4] My paper utilizes the concept of limits to arbitrage in a less direct way. Instead of focusing on the agency issue and risks for the arbitragers, I mainly focus on the belief dynamics and the relative market impact of noise traders. When their wealth level is high, arbitragers effectively face more difficulties correcting the mispricing. In addition, by focusing on extrapolators, I can not only provide a specific belief pattern for noise traders but also document a salient impact of investor beliefs on the aggregate stock market.

This paper is also relevant to the return predictability literature. Previous studies have suggested that aggregate stock returns are predictable using price-scaled variables, such as the dividend-

[4] For a discussion for limits to arbitrage, see Barberis and Thaler (2003).

to-price ratio and the earning-to-price ratio (Fama and French (1988), Campbell and Shiller (1988), and Cochrane (2011)). Some researchers attribute the predictability of stock market returns to variations in investors' required returns. However, behavioral theoretical models attribute the predictability to mispricings induced by investors' biased beliefs (Barberis et al. (2015), Hirshleifer, Li, and Yu (2015), and Jin and Sui (2017)). Bacchetta, Mertens, and van Wincoop (2009) document that the predictability of excess returns is often associated with the predictability of expectational errors, and this conclusion holds true for a broad set of asset classes, including the stock, foreign exchange and bond assets. Moreover, Cassella and Gulen (2015) empirically investigate the extent to which biased beliefs can help explain the observed predictability in the data. In this paper, I connect investor sentiment with return predictability through the time-varying impact of extrapolators and find empirical support for this result.

This paper also contributes to the literature that examines the impact of investor sentiment. Baker and Wurgler (2006) construct the investor sentiment index by directly using the first principal component of important market indicators, such as volume and equity share issuance, and demonstrate that investor sentiment has a large impact on the cross-section of stock returns. Baker and Stein (2004) propose an investor sentiment index based on market liquidity, and they show that it has predictive power for future market returns. Stambaugh, Yu, and Yuan (2012), who also use the sentiment index provided by Baker and Wurgler (2006), find that overpricing is more prevalent than underpricing when market-wide sentiment is high. However, most of these studies construct investor sentiment using a "top down" approach, which employs reduced-form variations in investor sentiment over time. By contrast, this paper uses a "bottom up" approach, and it focuses on the belief formation of investors. My focus on the microfoundations of the variation in investor sentiment allows me to shed new light on the dynamic patterns of the investor sentiment index. Like the "top down" literature, my analysis shows that investor sentiment has a large impact on the aggregate stock market.

Finally, this paper sheds light on important patterns in economic activities. First, many studies document a strong cyclicality of debt accumulation. Reinhart and Rogoff (2009) demonstrate that household debt accumulation speeds up during market booms and often leads to severe financial crashes. He and Krishnamurthy (2008) also propose a model that generates a procyclical leverage ratio that is based on financial intermediations. This model also helps generate a procyclical leverage ratio. However, in contrast to previous studies, in my model the procyclicality arises from extrapolation. During market booms, extrapolators become overly optimistic because they extrapolate the past returns and, consequently, they buy more risky assets. Conversely, during recessions, extrapolators become overly pessimistic and, therefore, they have a low leverage level. Second, extrapolation also helps explain the nagative association between household leverage and future consumption growth documented in Mian and Sufi (2009). Extrapolation induces investors to make unreasonable investment decisions that lead to a future decrease in wealth, which pushes down consumption growth rate.

Outline: This paper is organized as follows. In Section II, I document some dynamics of belief-based investor sentiment and identify the group of investors who are more susceptible to extrapolation. In Section III, I construct a behavioral model that incorporates both extrapolation and the wealth dynamics of extrapolators. Then I derive several model predictions about the time-varying impact of investor sentiment. In Section IV, I examine these model predictions via both simulation and data analysis. In Section V, I examine the role of extrapolation by considering a rational model as the benchmark. Section VI summarizes results and proposes directions for future research.

II. Motivating Facts

The difficulty of investigating the impact of belief-based investor sentiment lies both in how to correctly measure investor beliefs and how to measure its overall market impact. To provide insights into the dynamics of belief-based investor sentiment, I resort to investor expectation surveys which directly asks investors about their beliefs. In my later analysis, I will use two terms, belief-based investor sentiment and survey measurement of investor sentiment, interchangeably whenever there is no confusion. Further, relying on survey measurements of investor sentiment, I identify one specific group of investors who tend to extrapolate so that I can measure the market impact of extrapolators properly.

A. Investor Sentiment Dynamics

In recent decades, researchers have made progress in understanding investor expectations by analyzing survey evidence. In most of the existing investor surveys, respondents are asked about their expectations on future market returns, ranging from six to twelve months. Compared to other measurements of investor expectations, survey measurements are more direct in extracting investor belief information.

In this paper, I mainly use the Gallup survey which measures individual investors' expectations of the U.S. stock market over the next twelve months.[5] It is conducted monthly between 1996 and 2011, but there are some gaps in later years especially between November 2009 and February 2011 when the survey was discontinued. To extract investor expectations, in each month, Gallup survey asks participants one *qualitative* question: whether they are "very optimistic", "optimistic", "neutral", "pessimistic", "very pessimistic" about stock returns over the next twelve months. With the percentage of each response in the collected survey answers, Gallup reports a *qualitative* investor expectation series to measure investor expectations in the market:

$$Gallup = \%Bullish - \%Bearish, \tag{1}$$

where "Bullish" is defined as either "very optimistic" or "optimistic" and "Bearish" is defined as either "pessimistic" or "very pessimistic". This qualitative time series helps us understand the dynamics of investor sentiment in the market. Moreover, Gallup survey also asks more precise *quantitative* questions about investors' perceived expected returns, although only for a shorter sample. Specifically, between September 1998 and May 2003, Gallup asks participants to give an estimate of the percentage return they expect for the stock market over the next year. Therefore, as long as participants in the Gallup survey answer quantitative and qualitative questions in a consistent way, I can effectively get quantitative estimations investor expectation series by rescaling qualitative Gallup investor series with projection method.[6] This projection method also helps me to transform qualitative series in other investor expectation surveys to a meaningful quantitative basis.

However, there were two main concerns about the survey data. One concern is that survey evidence is imprecisely measured and thus is noisy. The other is that survey respondents may be

[5] I use Gallup survey measurement to measure investor sentiment since Gallup mainly surveys individual investors who are more likely to extrapolate. Moreover, Gallup survey has a quantitative measurement of investor expectations to facilitate my analysis.

[6] Carlson and Parkin (1975) propose a method to generate average expectations from categorical survey data. As pointed in Greenwood and Shleifer (2014), this method has almost no impact on the investor expectation time series.

confused by the sophisticated questions and therefore could not provide pertinent answers. Fortunately, there are recent developments that show the validity of the investor survey information. Greenwood and Shleifer (2014), among other findings, show that (1) information contained in different surveys reflects similar patterns and (2) the reported investor expectations in the surveys are largely consistent with investors' behaviors (See Figure 2).[7] Their findings indicate that survey measures of investor expectations are not meaningless noise but represent widely shared beliefs about future returns in the stock market. With these validations from previous literature, I report the following observations to motivate my model.

Observation 1. *Survey measurement of investor sentiment is positively associated with the past returns in the aggregate stock market.*

Observation 1 is the main message in Greenwood and Shleifer (2014), and is a restatement of extrapolation: investors in the surveys over-extrapolate recent returns when forming their expectations in their minds. Formally, I use *extrapolators* to refer to these investors. Therefore, past good returns tend to make extrapolators overly optimistic while past bad returns make them overly pessimistic. For the underlying psychological mechanisms of extrapolation, there are several candidate theories, including representativeness. Kahneman and Tversky (1972) define representativeness as "the degree to which an event (i) is similar in essential characteristics to its parent population, and (ii) reflects the salient features of the process by which it is generated".[8] With representativeness, extrapolators might mistakenly treat a sequence of good (bad) recent returns as a salient feature of the whole distribution of returns, which leads to an over-extrapolation. Although the fact that extrapolators overweight information in the recent returns is commonly documented in recent empirical studies (Amromin and Sharpe (2013), Bacchetta et al. (2009) and Greenwood and Shleifer (2014)), the source of over-extrapolation still remains an important open question.

The belief-based investor sentiment in the surveys is consistent with most of the anecdotal fluctuations of investor sentiment—that investor sentiment rise rapidly during booms and decrease

[7]For result one, the authors compare survey sources from the American Association of Individual Investors (AA), Gallup, Graham and Harvey, Investors' Intelligence newsletter expectations, Michigan Survey and Shiller, and document strong correlations between each survey. For result two, the authors examine the reported investor expectations and the investor mutual fund flows, and find two time-series are highly synchronized.

[8]For other references, also see Tversky and Kahneman (1971) and Tversky and Kahneman (1975).

during crash episodes—in the market. In Figure 1, I plot the Gallup survey measurement of investor sentiment, ranging from 1996:12 to 2011:09, with the backdrop of shaded NBER recessions. During the "Internet Bubble" episodes in mid-2000, Gallup survey measurement of investor sentiment rises to its peak but drops dramatically after the burst. Similarly, Gallup survey measurement rebounds to the peak before the 2007 financial crisis and declines significantly after the market index fell in 2008.

[Place Table 1 about here]

To further reveal the close connections between investor sentiment in the survey and extrapolation, I construct a new investor sentiment variable called "Psentiment". Previous studies on the belief patterns in the survey indicate that when forming beliefs on expectations of returns, extrapolators put a decaying weight on the realized returns in the past. For example, Barberis et al. (2015) use the non-linear formula in equation (2) to estimate the weighting scheme of extrapolators, with the survey measurements of investor sentiment as the dependent variables and ψ represents for the weighting scheme. $R_{t-(s+1)\Delta t,t-s\Delta t}$ measures the past realized returns within one interval Δt. Using data of quarterly frequency ($\Delta t = 1/4$), they get the estimated ψ of 0.44, implying that the realized returns one year ago are only half as important as the most recent return.

$$
\begin{aligned}
\text{Expectation}_t &= a + b\Sigma_{s=0}^{\infty}\omega_s R_{t-(s+1)\Delta t,t-s\Delta t}, \\
\omega_s &= \frac{e^{-\psi s\Delta_t}}{\Sigma_{k=0}^{n} e^{-\psi k\Delta_t}},
\end{aligned} \tag{2}
$$

The constructed investor sentiment variable, Psentiment, is purely based on extrapolation and I use it for robustness check—if investor beliefs in the surveys are mainly driven by extrapolation, then Psentiment and Gallup survey measurement of investor sentiment should yield similar results.

Observation 2. *Survey measurement of investor sentiment tend to revert to its mean: high investor sentiment revises downwards in the future while low investor sentiment revises upwards in the future.*

To empirically test the pattern in Observation 2, I run the following regression:

$$SR_{t+1}[\text{Sent}_{t+N} - \text{Sent}_{t+1}] = c + d\text{Sent}_t + u_t, \tag{3}$$

where on the left hand side $SR_{t+1}[\text{Sent}_{t+N} - \text{Sent}_{t+1}]$ measures the investor sentiment revision (SR for short) over future $N - 1$ horizons, Sent_t represents the investor sentiment at time t, and u_t on the right hand side is the corresponding residual at time t. The results for equation 3 is reported in Table II.

[Place Table II about here]

The negative coefficient d supports the mean-reversion pattern in Observation 2. Using the Gallup survey, one standard deviation increase implies an investor sentiment revision of 14.2% within one quarter and a revision of 62.4% within four quarters. Similarly, with the constructed investor sentiment proxy Psentiment, one standard deviation decrease implies an investor sentiment revision of 6.4% with one quarter and a revision of 32.9% within four quarters. The coefficients remain statistically significant in general.

The reversal property in expectations in Observation 2 is largely due to the fact that investors tend to overreact to the information, and therefore it reflects the excessive optimism and pessimism on expectations of returns in the surveys. When investor sentiment is high, investors perceive higher returns going forward. However, the objective return distributions observed by econometricians remain unchanged – investors get constantly disappointed by future realized returns. As a result, investor sentiment revises downwards in the future. When investor sentiment is low, investors get constantly surprised by future realized returns, and investor sentiment bounce upwards.

Reversal is a general pattern in economic studies. For instance, Greenwood and Hanson (2013) document a systematic reversal in bond spreads. Bordalo, Gennaioli, and Shleifer (2016) document predictable reversals in the credit market because credit spreads overreact to news. Reversals in investor expectations are also common and are not limited to expectations of stock returns. For instance, Gennaioli, Ma, and Shleifer (2016) document a salient reversal pattern for CEOs' expectations about their company's earnings growth, and interpret the reversal as a result of over-extrapolating past earnings growth rate. López-Salido, Stein, and Zakrajšek (2017) also links reversals to overreaction, and suggest that a period of excessive investor optimism is followed by a reversal in the credit market, a term they refer to as "unwinding of investor sentiment". The reversal patterns capture important properties of investor belief dynamics.

B. *Wealth Level of Extrapolators*

The documented patterns in Observations 1 and 2 provide evidence on the dynamics of investor beliefs. However, investor beliefs alone could not provide an answer to the question of how belief-based investor sentiment influences the market: as shown in Table III, Gallup survey measurement of investor sentiment does *not* have significant predictive power for future stock market returns. This result seems puzzling at the first glance since previous studies have shown that the extrapolation pattern in surveys reflect market-wide investor expectations—intuitively, the predictive pattern should be strong. One important element that seems missing is the wealth level of extrapolators: after all, extrapolators with extreme sentiment levels but with trivial wealth will have a limited impact on asset dynamics. Therefore, I also examine the wealth level of extrapolators.

To measure the wealth level of extrapolators, it is important to understand which group of investors are more susceptible to extrapolation. In the finance literature, a common way to categorize investors is to divide them into individual investors and institutional investors. Moreover, in the literature, individual investors are often believed to be less sophisticated and more vulnerable to psychological biases, a term formally defined as "dumb money" effect. And empirical evidence is prevailing. For instance, Odean (1999), Barber and Odean (2000) and Barber and Odean (2001) present extensive evidence that individual investors suffer from biased-self attribution, and tend to have wealth-destroying excessive trading. Frazzini and Lamont (2008) use mutual fund flows as a measure of individual investor sentiment for different stocks and find that high sentiment predicts low future returns.[9]

A reasonable proxy for individual investors exists in the Federal Reserve's Z.1 Statistical Release ("Financials Accounts of the United States"), which reports balance sheet information for different sectors of the economy at a quarterly frequency, including the Households and Nonprofit Organizations Sector (HNPO sector). The HNPO sector contains aggregated information about

[9] For other studies supporting the "dumb money" effect, see Barber, Lee, Liu, and Odean (2008), Sapp and Tiwari (2004).

individual investors.

More importantly, as shown in the recent studies, individual investors in the HNPO sector tend to extrapolate. For example, a rigorous examination is reported is Yang and Zhang (2017). Specifically, they document that (1) when investor expectations in the surveys are high, investors in the HNPO sector tend to increase their investment in the stock market; and (2) their investment choices negatively predict future stock market returns. These results together show that investors in the HNPO sector can serve as a reasonable proxies for extrapolators. Moreover, in my empirical analysis, I use the total financial wealth to proxy for the wealth level of extrapolators.[10]

C. *Interaction Effect between Investor Sentiment and Wealth Level*

Investor sentiment should impact market strongly especially when the wealth level of extrapolators is high. In other words, there should be a strong interaction effect between investor sentiment and the wealth level of extrapolators in explaining asset valuations. To formally test this hypothesis and to motivate my model setting, I run the following regression:

$$R^e_{t+N} = a + b\text{Sent}_t + cW_t/D_t + d\text{Sent}_t \times W_t/D_t + \epsilon_t. \tag{4}$$

In my regression, R^e_{t+N} represents the excess returns of the aggregate stock market during the future N months. In addition, I use investor sentiment in the Gallup survey and Psentiment, respectively, to proxy for Sent_t, investor sentiment, and use the total financial assets of HNPO sector at time t as the empirical proxy for the wealth level W_t. Since W_t is non-stationary, I get the series of real dividend D_t from Shiller's website and get the normalized wealth level of W_t/D_t. My sample is at a quarterly frequency and spans from 1996:12 - 2011: 9.[11] For robustness check, I report predictive regressions over one to six quarters.

[Place Table IV about here]

The results reported in Table IV not only confirm the strong interaction effect between investor sentiment and the wealth level in determining the asset prices, but also point to the usefulness

[10] As long as the HNPO sector captures the general properties of extrapolators, it will help understand the impact of investor sentiment on the market.

[11] The Federal Reserve's Z.1 Statistical Release is published at a quarterly frequency. The sample period mainly goes with Gallup surveys.

in explaining asset *mispricings*- the strong negative coefficient for the interaction term indicates investor sentiment leads to asset overvaluations and undervaluations when the wealth level is high. When investor sentiment is high, the market is highly overvalued due to the high irrational demand from extrapolators and therefore future returns are low; if investor sentiment is low, the future returns are high since the market is undervalued. In addition, compared to the univariate regression using investor sentiment variable, the conditional predictability model has a significantly higher adjusted-R^2. For instance, at the annual horizon, the goodness of fit for the predictive regression using investor sentiment alone is around 0.010, which is small compared to 0.039 in the conditional predictive regression. The increased adjusted-R^2 supports the importance of the interaction effect in explaining the market mispricing. Therefore, I get the following observation:

Observation 3. *Investor sentiment strongly connects with market mispricing through the market impact of extrapolators.*

Observation 3 provides strong evidence for the relation between the market mispricing, investor sentiment and the wealth level of extrapolators. Therefore, for a model that focuses on the impact of investor sentiment and market mispricing, the wealth level of extrapolators should also be incorporated.

III. Behavioral Model

In this section, I build a behavioral model that focuses on the investor sentiment of extrapolators. Specifically, following the patterns of investor sentiment documented in section II, I introduce extrapolation into my model. With extrapolation, the behavioral model essentially captures the overvaluations and undervaluations in the market and shed light on the time-varying impact of investor sentiment on the equilibrium asset price.

A. The Economy

I consider a continuous-time economy with two types of assets: one risk-free asset with elastic supply curve and a constant rate r, and one risky asset with a fixed per-capita supply of one. Due to extrapolation, extrapolators perceive a biased growth rate of the risky asset, which is different from the true growth rate observed by the outside econometricians.

The risky asset is a claim to the underlying dividend process D_t which, under the true probability measure, follows a geometric Brownian motion and can be generically written as:

$$dD_t/D_t = g_D dt + \sigma_D d\omega_t. \tag{5}$$

Therefore, the underlying dividend process is governed by the growth rate of g_D and volatility σ_D, which both are positive exogenous parameters. The dividend process is driven by ω_t, a one-dimensional Weiner process under the true probability measure observed by outside econometricians. The equilibrium price P_t for the dividend claim evolves as

$$dP_t/P_t = g_{P,t} dt + \sigma_{P,t} d\omega_t. \tag{6}$$

Due to extrapolation, the true growth rate $g_{P,t}$ is different from the perceived growth rate by extrapolators. The growth rate $g_{P,t}$ and volatility term $\sigma_{P,t}$ are both endogenously determined in the equilibrium.

B. Investors

There are two types of investors in the behavioral model: extrapolators and fundamental investors. Extrapolators are the focus of this paper: their belief formation are subject to psychological heuristics and therefore are misspecified. Fundamental investors, on the other end, serves as the counteracting forces in the market and trade aggressively whenever asset prices deviate from fundamental values. Similarly, I assume that fundamental investors make up a fraction of $1 - \mu$ and extrapolators make up a fraction of μ.

B.1. Extrapolative Beliefs

The salient properties about investor sentiment in the surveys – that investors form their expectations based on past realized returns and that their sentiment reverts quickly to its mean – motivate my theoretical settings for investor sentiment. Different from asset pricing models with rational expectations, extrapolators in the behavioral model make systematic errors about future market returns. In order to capture the misspecified beliefs for investors, I propose a mental model.[12] Specifically, I assume extrapolators perceive the following process for the market price, with the perceived growth rate $\hat{g}_{P,t}$ as an affine function of a latent state variable S_t that essentially captures the excessive optimism and pessimism in extrapolators' mind:

$$dP_t/P_t = \hat{g}_{P,t}dt + \sigma_{P,t}d\omega_t^e,$$

$$\hat{g}_{P,t} \equiv [(1-\theta)\bar{g}_{P,t} + \theta S_t]. \tag{7}$$

Here $\bar{g}_{P,t}$ represents the equilibrium growth rate of risky asset in a benchmark economy where each investors have correct beliefs. Parameter θ measures to which extent extrapolators deviate from their beliefs in the rational benchmark. When $\theta = 0$, I get the rational benchmark case with a drift term $\hat{g}_{P,t}$.[13] Notation-wise, any quantities with a hat sign or superscript e represent variables perceived by extrapolators.

Next, I introduce microfoundations for this key latent state variable S_t. Specifically, I assume that from the extrapolators' perspective, the expected growth rate on the risky asset is an affine function of a mental variable $\tilde{\mu}_{P,t}$, and the extrapolators mistakenly believe that this mental variable is governed by a regime-switching process between high and low states μ_H and μ_L, with switching densities χ and λ:

$$\begin{array}{r c} & \begin{array}{cc} \tilde{\mu}_{P,t+dt} = \mu_H & \tilde{\mu}_{P,t+dt} = \mu_L \end{array} \\ \begin{array}{r} \tilde{\mu}_{P,t} = \mu_H \\ \tilde{\mu}_{P,t} = \mu_L \end{array} & \left(\begin{array}{cc} 1-\chi dt & \chi dt \\ \lambda dt & 1-\lambda dt \end{array} \right). \end{array} \tag{8}$$

Motivated by the fact that investors in the survey over-extrapolate past returns when forming their expectations, I assume that extrapolators in my model update their estimate of the mental variable $\tilde{\mu}_{P,t}$ by looking at the past realized returns. Therefore, μ_H and μ_L reflect extrapolators' subjective perceptions on the risky asset growth rate, and λ and χ represent the speed extrapolators update their perceptions on the risky asset growth rate based on past realized returns. Given this mental model, the latent state variable S_t is the Bayesian inference of the mental variable $\tilde{\mu}_{S,t}$. Formally,

[12] A similar model is used in Jin and Sui (2017).
[13] For a detailed solution for $\hat{g}_{P,t}$, see appendix.

I can write the latent state variable as $S_t \equiv \mathbb{E}^e[\tilde{\mu}_{P,t}|\mathscr{F}_t]$, where $\mathscr{F}_t$ is the perceived probability measure based on the filtration of the price process P_t. My later examination on extrapolators' belief structure helps me justify the magnitude of these belief parameters.

Therefore, in my model, investor sentiment corresponds to the perceived growth rate of extrapolators, $\hat{g}_{P,t}$. Throughout my model, I assume that each extrapolators are subject to the identical underlying mental model and the same degree of extrapolation. In other words, latent state variable θ reflects the *consensus* extrapolation level among extrapolators.

By applying the optimal filtering theorem (Liptser and Shiryaev (2013)), I obtain the dynamics for the latent state variable S_t:

$$dS_t = \mu_S dt + \sigma_S d\omega_t^e, \tag{9}$$

where

$$\mu_S = \lambda\mu_H + \chi\mu_L - (\chi + \lambda)S_t, \tag{10}$$

$$\sigma_S = \sigma_{P,t}^{-1}\theta(\mu_H - S_t)(S_t - \mu_L), \tag{11}$$

$$d\omega_t^e = \frac{dP_t}{P_t} - [(1-\theta)\bar{g}_{P,t} + \theta S_t]dt. \tag{12}$$

This mental model captures two salient features of investor sentiment in the surveys. First, under the mental model, the latent state variable S_t is driven by the perceived Brownian shock $d\omega_t^e$, which strongly depends on the past realized returns. Increases in past realized returns $\frac{dP_t}{P_t}$ push up the perceived Brownian shock, which further increases the latent state variable S_t and the perceived growth rate of the risky asset $\hat{g}_{P,t}$. Therefore, the mental model naturally leads to an extrapolation pattern: high returns in the past push up investor sentiment and low past returns make extrapolators pessimistic.

Second, the mental model embodies the fact that investor sentiment tends to revert to its mean. When the latent state variable is high, extrapolators mistakenly perceive a high level of growth rate for the risky asset. However, the objective probability measure remain unchanged. Unless extremely good shocks arrive, they will be constantly disappointed by the perceived Brownian shocks $d\omega_t^e$ in the future. As a result, the latent variable will quickly revert downwards. Similarly, if the latent state variable is low and extrapolators perceive a low level of growth rate, extrapolators will meet with relatively large perceived Brownian shocks, pushing up the latent state variable back to its mean.

The underlying mechanism for the reversal of investor sentiment in my model is different from previous "Natural Expectations" models in Fuster, Hebert, and Laibson (2011). In Fuster et al. (2011), the long-term reversal are exogenously assumed and errors in expectations arise because agents fit a simpler AR(1) model to the data. As a comparison, in my model, the mean-reversal process is endogenous—investors overweight recent information and this overreaction leads to reversals. Such endogenous reversal pattern in investor beliefs helps understand asset mispricings and predictability of returns generated in this model.[14]

Moreover, due to the mental model, the perceived probability measure by extrapolators is different from the objective probability measure observed by outside econometricians.[15] In other words, extrapolators have misspecified but self-consistent beliefs, and they fail to realize that their perception is biased. The true and perceived probability measure can be connected using the following equation:

$$d\omega_t^e = (g_{P,t} - \hat{g}_{P,t})/\sigma_{P,t} dt + d\omega_t. \tag{13}$$

Under the probability measure of extrapolators, the perceived dividend process follows

$$dD_t/D_t = \hat{g}_{D,t} dt + \sigma_D d\omega_t^e. \tag{14}$$

where $\hat{g}_{D,t}$ is the perceived dividend growth rate by extrapolators.

In my model, $g_{P,t}$, $\hat{g}_{P,t}$ and $\hat{g}_{D,t}$ are endogenous variables determined in the equilibrium. As a comparison, the volatility variables $\sigma_{P,t}$ and σ_D remain unchanged. This is because extrapolators can always calculate the volatilities by calculating the quadratic variations of the stochastic process. All the quantities together constitute two mutually coherent probability measurements.

Finally, it is worth pointing out that, there is a two-way feedback loop in my model: asset prices

[14]The pattern that investors tend to overweight recent information has been incorporated in other models as well. For instance, Bordalo et al. (2016) focus on investors' belief formation of credit spreads, and proposed a mechanism in which investors overreact to recent news.

[15]The fact that investors have biased beliefs is widely empirically documented in the literature. For example, using option data, Barone-Adesi, Mancini, and Shefrin (2016) document excessive optimism and pessimism for investors in the U.S. stock market.

influence investor expectations due to extrapolation, and changes in investor expectation will in turn drive asset prices. From a theoretical perspective, a model with extrapolation on past returns is usually difficult to solve, since the return process is an endogenous quantity determined in the equilibrium. In this paper, I overcome this difficulty by solving a system of partial differential equations.

B.2. Wealth Process of Extrapolators

To better connect with later analyses on the time-varying impact of investor sentiment, I introduce logarithmic utilities for extrapolators to capture the dependence of demand on their wealth level. Moreover, each extrapolator, indexed by superscript j, is infinitesimal and therefore acts as price-taker in the market. Specifically, extrapolator j maximizes an additively separable logarithmic utility function under an infinite time horizon and with a time-preference parameter ρ

$$\max_{\{C^j_{t+s}\}_{s\geq 0}} \mathbb{E}^e_t[\int_t^{\infty} e^{-\rho s} \ln C^j_s ds] \tag{15}$$

subject to

$$dW^j_t = -C^j_t dt + rW^j_t dt + \alpha^j_t W^j_t [\hat{g}_{P,t} dt + \frac{D_t}{P_t} dt - rdt + \sigma_{P,t} d\omega^e_t]. \tag{16}$$

where W^j_t represents the wealth level of extrapolators, C^j_t the optimal consumption choice and α^j_t the optimal risk exposure for extrapolator j. $\mathbb{E}^e_t$ represents the expectation operator under the perceived probability measure, which is biased due to extrapolation.

With the logarithmic utility, extrapolators have wealth-dependent absolute risk aversion: as their wealth declines to zero, extrapolators become infinitely risk-averse. Consequently, as their wealth shrinks, they will liquid the risky asset to prevent their wealth from becoming zero.[16] In addition, the infinitely risk-aversion level at zero wealth level also prevents extrapolators from bankruptcy. Therefore, they can borrow money at risk-free rate irrespective of their wealth level.

Following the standard Merton's approach (Merton (1971)), we get the optimal consumption and portfolio rule for extrapolators:

PROPOSITION 1: *Extrapolators with the objective function in equation (15) and wealth process in*

[16]The existence of fundamental investors guarantees they can liquidity their risky asset position at any time.

equation (16) will optimally consume and invest at time t according to the following strategies:

$$C_t^j = \rho W_t^j \tag{17}$$

and

$$\alpha_t^j = \frac{\hat{g}_{P,t}(x_t, S_t) + l^{-1}(x_t, S_t) - r}{\sigma_{P,t}^2(x_t, S_t)}. \tag{18}$$

where the price to dividend ratio $l \equiv \frac{P_t}{D_t}$ and the risky asset volatility $\sigma_{P,t}$ both depend on the wealth to dividend ratio $x_t \equiv \frac{W_t}{D_t}$ and the latent state variable S_t. In other words, the economy depends on state variables x_t and S_t.

Proof: See Appendix C.

Proposition (1) documents several salient features of logarithmic utilities. First, the optimal consumption strategy for extrapolators is proportional to their wealth level, with adjustment of the time-preference parameters ρ. Second, the optimal risky portfolio choice is myopic, in a sense that extrapolators do not hedge against changes in the future investment set. Therefore, their optimal risky exposure purely depends on the perceived risk premium and the instantaneous volatility rate. Third, the total dollar demand is also proportional to their wealth. Therefore, their wealth level, by and large, determines their market impact on the equilibrium asset prices—a key property that drives my model implications.

In the equilibrium, the risky exposure of extrapolators depends on their perceptions of equity premium, $\hat{g}_{P,t} + l^{-1} - r$. After observing a sequence of high (low) returns, extrapolators become overly optimistic (pessimistic) and increase their risky asset exposure. In cases when their market impact is high, asset tends to become overvalued (undervalued).

More importantly, the optimal strategy in equation (18) and the wealth dynamics in equation (16) together provide novel insights on how investor sentiment influences the wealth dynamics of extrapolators. When investor sentiment is high, extrapolators perceive high equity premium and therefore lever up to buy risky asset. However, since investor sentiment reverts to its mean, extrapolators demand less risky asset in the future, which leads to decreases in the asset prices and their wealth level, therefore generating negative returns. In the mean-reversion process, the wealth dynamics of extrapolators also play a role: investor sentiment leads investors to take excessive exposure to the risky asset and therefore suffer from wealth decrease. Decline in the wealth level of extrapolators makes the mispricing easier to correct.

B.3. Fundamental Investors

Next, I describe the demand function for fundamental investors. I assume that fundamental investors care only about the difference between the current risky asset prices and their fundamental values, hence they construct their risky asset exposure based on the deviation of the asset prices from its fundamental values. Following this assumption, the per capita demand of the risky asset for fundamental investors follows

$$Q_t = (P_{F,t} - P_t)/k, \tag{19}$$

where $P_{F,t} = \frac{D_t}{r-g_D}$ is the fundamental value and k is a constant. This linear demand structure is standard in the literature (for example Xiong (2001)), and I provide a micro-foundation for it in the appendix A. Intuitively, when $P_{F,t}$ is higher (lower) than its current price, fundamental investors expect profits when asset prices reverts back to the fundamental value P_t. Therefore, they have a strong demand to short (buy) the risky asset.

Fundamental investors are prevailing in the financial market. There are many investors who truly follow strategies that focus on the long-term profit and apply fundamental analysis in their investment. For instance, Abarbanell and Bushee (1997) document that investors incorporate financial statement information and make fundamental analysis when making their investment decision. In addition, there are many mutual funds who set their investment objective as to achieve long-term growth of capital and income, such as the "Fundamental Investor" fund managed by Capital Group. Moreover, such fundamental analysis really brings abnormal returns to their portfolio. For instance, Piotroski (2000) document fundamental analysis can increase annual returns by at least 7.5%.

Several observations about the total dollar demand in equation (19) are worth noting. First, for fundamental investors, the total dollar demand is independent of their wealth level, rather, it only depends on the difference between the current risky asset prices and their fundamental values. When asset prices are highly overvalued, fundamental investors anticipate a higher return from holding the risky asset and, as a result, their total dollar demand increases. Conversely, when asset prices are undervalued, fundamental investors lean against the wind and trade actively to push prices upward. Moreover, the extent to which fundamental investors can correct the mispricing also depends on the overall market impact of the extrapolators: if the wealth level of extrapolators is low, fundamental investors can correct mispricing more easily. This demand structure of fundamental investors proves to be useful in generating novel implications on the predictive pattern of investor sentiment.

Second, in my model, fundamental investors act as aggressive arbitragers and they jump into arbitrage activities *whenever* mispricing occurs – different from fully rational investors, fundamental investors in my model are the counteracting forces in the market. In other words, irrespective of their beliefs, fundamental investors have a specific demand function due to unspecified factors such as financial frictions or preference shocks.

C. Equilibrium

Definition of Equilibrium:

An equilibrium in the behavioral model satisfies the following conditions:

i) extrapolators maximize their objective function in equation (15) subject to their wealth process in equation (16) under their subjective probability measure induced by extrapolation;

ii) fundamental investors invest in the risky asset according to their demand function in equation (18);

iii) risky asset market clears

$$\mu \alpha_t W_t + (1 - \mu) Q_t = P_t. \tag{20}$$

To solve for the equilibrium, I face with a fixed-point problem: the optimal demand of extrapolators depends on the instantaneous equity premium and volatilities of the risky asset, which further depends on the total demand in the equilibrium. For the behavioral model, I focus on the symmetric equilibrium where each extrapolator endows with the same level of wealth and follows identical strategies and solve for a fixed-point problem. The equilibrium depends both on the wealth to dividend ratio x_t and the latent state variable S_t. Therefore, I rely on solving a partial differential equation to obtain the solution for the fixed-point problem. As verified later, the price to dividend ratio is monotonically increasing in both x_t and S_t.

After getting the optimal strategy for extrapolators, now I consider a fixed-point problem and solve the Markovian system to get an solution of the price-dividend ratio $l(S_t, x_t)$. To be specific, I focus on an equilibrium, where each extrapolator j follows the same strategies based on the return extrapolation. By using aggregation and the market clearing condition, I can reach the equilibrium as follows:

PROPOSITION 2: *In the symmetric equilibrium, the price-dividend ratio l is a function of current states S_t and x_t, and satisfies the following partial differential equation:*

$$\frac{c_0 l - c_1}{x} = \frac{\hat{g}_{P,t} + l^{-1} - r}{\sigma_{P,t}^2}, \tag{21}$$

where σ_P is also a function of S_t and x_t and satisfies

$$\sigma_{P,t} = \frac{(\frac{l_x}{l}x - 1)\sigma_D - \sqrt{(\frac{l_x}{l}x - 1)^2\sigma_D^2 - 4(\frac{l_x}{l}(c_0 l - c_1) - 1)\frac{l_S}{l}\theta(\mu_H - S)(S - \mu_L)}}{2(\frac{l_x}{l}(c_0 l - c_1) - 1)}, \tag{22}$$

and $l(S_t, x_t)$ satisfies the following boundary conditions

$$\lim_{x_t \to 0} l = \frac{c_1}{c_0}, \tag{23}$$

$$\lim_{x_t \to 0} = \lim_{x \to 0} \frac{\hat{g}_{P,t} + \frac{c_1}{c_0} - r}{\sigma_D^2}, \tag{24}$$

and

$$\lim_{x_t \to \infty} l = (r - \hat{g}_{P,t})^{-1}, \tag{25}$$

where $c_0 = \frac{k+1-\mu}{k\mu}$ and $c_1 = \frac{1-\mu}{k\mu(r-g_D)}$ are both constants.

In addition, under the perceived probability measure, the dynamics of x_t follow

$$dx_t = \hat{g}_{x,t}(S_t, x_t)dt + \sigma_{x,t}(S_t, x_t)d\omega_t^e, \tag{26}$$

where

$$\begin{aligned}\hat{g}_{x,t}(S_t, x_t) =& x_t(r - \rho + \alpha_t[\hat{g}_{P,t} + l^{-1} - r] - \hat{g}_{D,t} + \sigma_D^2 - \alpha_t\sigma_D\sigma_{P,t}) \\ \sigma_{x,t}(S_t, x_t) =& x_t(\alpha_t\sigma_{P,t} - \sigma_D).\end{aligned} \tag{27}$$

Proof: See Appendix C.

For the boundary conditions, when the wealth to dividend ratio goes to zero, the fundamental investors dominate the market and I get a constant price-dividend ratio in equation (23). Moreover, in this case, the partial derivative of l with respect to S, $l_S(S, 0)$, equals to zero because extrapolators' total demand is zero irrespective of the level of the latent state variable S_t. On the other hand, when x_t goes to infinity, in order to clear the market, extrapolators hold a risky asset position close to zero. By the portfolio expression in equation (18), we get the expression (25). Expression (24) follows naturally from equation (23).

In solving the ordinary equation in proposition (2), I resort to a projection method with Chebyshev polynomials. Compared to the range of the wealth to dividend ratio of $[0, \infty]$, the required domain for Chebyshev polynomial is $[-1, 1]$. Therefore, I apply the following monotonic transformation for the wealth to dividend ratio

$$z_t = \frac{x_t - \xi}{x_t + \xi}, \tag{28}$$

where ξ is a positive constant. Along with the transformed wealth to dividend ratio, I also transform the latent state variable S into a new variable y that lies between $[-1, 1]$:

$$\begin{aligned}y_t =& aS_t + b, \\ a =& \frac{2}{\mu_H - \mu_L}, \quad b = -\frac{\mu_H + \mu_L}{\mu_H - \mu_L}.\end{aligned} \tag{29}$$

When the wealth to dividend ratio goes to infinity, the transformed wealth to dividend ratio goes to 1. If the wealth to dividend ratio shrinks to zero, the transformed wealth to dividend ratio goes to -1. Similarly, y_t goes to 1 when S_t goes to its upper bound μ_H, and goes to -1 when S_t goes to its lower bound μ_L.

D. *Calibrated Model Solution*

In this section, I report the main numerical solutions. To get a reasonable model solution, I choose both asset and utility parameters that are consistent with the empirical literature. For example, I set g_D to be 1.5% and σ_D to be 10%, which are commonly used in the asset pricing literature. In addition, I set $r = 4\%$ for the risk-free rate. For utility parameters, I choose the time-preference factor $\rho = 2\%$. For other parameters that have no empirical counterparts, such as μ and k, I take a neutral stand and impose a value of $\frac{1}{2}$. For belief parameters, I set $\mu_H = 0.03$, $\mu_L = -0.06$, $\chi = \lambda = 10\%$, $\theta = 0.5\%$. A complete set of parameter values are reported in Table VI. I report the numerical solutions in Figure 5.

D.1. Price-Dividend Ratio

The upper-left panel reports the price to dividend ratio $l(S_t, x_t)$. In general, $l(S_t, x_t)$ is a monotone function both in the transformed wealth to dividend ratio z_t and the latent state variable y_t. First, for most of z_t levels, a higher latent variable level S_t induces higher total dollar demand and therefore pushes up the price to dividend ratio. When $z_t = 0$, the price to dividend ratio $l(S_t, x_t)$ are constant values of 20: if extrapolators have no wealth, they will have no market impact on the equilibrium quantities. Second, with a fixed level of latent variable y_t, the dollar demand for the risky asset increases as z_t increases. With given parameters, the solution for $l(S_t, x_t)$ ranges from 20 to 35, which is largely within the reasonable range.

[Place Figure 5 about here]

D.2. Optimal Portfolio Choice

I report the optimal portfolio choice for extrapolators in the lower-right panel. As y_t increases along the axis, extrapolators perceive higher growth rate and therefore increase their position in the risky asset. As z_t increases from -1 to 1, in general, the optimal portfolio decreases in order to meet the market clearing condition. Together, I get the optimal strategies of extrapolators. It is worth noting that, when both z_t and y_t are low and extrapolators have low market impact and become pessimistic about future return growth, they can short the risky asset; in other situations, extrapolators hold the risky asset.

D.3. Volatility

The upper-right panel portrays the return volatility σ_P. In most of the cases, σ_P is larger than σ_D. When the wealth to dividend ratio x_t is high and the latent state variable S_t is slightly above its mean, the volatility σ_P reaches its maximum of 17.78%.

Along the latent variable axis, the volatility has a strong hump-shaped pattern due to the belief structure. Specifically, when the latent variable y_t goes to its upper or lower boundary, there is less uncertainty about which regime the underlying state belongs to. On the contrary, when y_t is in the middle region between the high and low bound, the uncertainty increases.

Moreover, increases in z_t in general increases the market impact of extrapolators. As a result, their beliefs have a stronger amplification effect on the exogenous shocks, which explains the increasing volatility along the z_t axis. However, when y_t reaches its maximum or minimum, the volatility is mainly determined by changes in z_t ratio. When y_t is at its maximum, the optimal portfolio is positive but less than one. In this case, a price drop caused by a negative dividend shock leads extrapolators to rebalance their portfolio by purchasing more of the risky asset, which reduces the volatility.[17] Conversely, when y_t is at its minimum, the optimal portfolio is, by and large, negative. In this case, extrapolators short the risky asset. When positive dividend shocks arrive, extrapolators' leverage ratio further decreases and induces them to buy more risky assets, which increases the volatility.

E. Two Model Predictions

E.1. Investor Sentiment and Market Mispricing

In this model, I simultaneously characterize investor sentiment and the wealth level within one unified model, which provides novel insights between investor sentiment and market mispricing. From a static perspective, the market clearing condition in equation (20) helps to identify when the market overvaluations and undervaluations would occur. When the wealth level of extrapolators is

[17] Since extrapolation simultaneously make extrapolators less willing to hold the risky asset, the portfolio-rebalancing effect is weaker than that in the rational benchmark model.

high, investor sentiment has large impacts on the market: with high investor sentiment, the asset price is largely overvalued; while if investor sentiment is low, the asset price is undervalued.

E.2. Investor Sentiment and Return Predictability

The dynamic feature of the behavioral model also sheds light on the connection between investor sentiment and the future asset price dynamics. With higher wealth level, a larger fraction of total asset demand will come from extrapolators. Conditional on high wealth level, high investor sentiment will push asset prices above their fundamental value. However, higher sentiment will revert to its mean quickly, since optimistic investors will more easily get disappointed by the future realized returns. Consequently, both investor sentiment and asset prices will decline in the future, generating negative returns. Conversely, conditional on high wealth level, low investor sentiment predicts a high returns going forward. This together implies a strong negative pattern for investor sentiment when wealth level is high.

Moreover, my model implies a novel predictive pattern of investor sentiment on future returns when the wealth level of extrapolators is low. In this situation, asset prices are mainly driven by fundamental investors and the sentiment of extrapolators reflects the direction of market correction. If extrapolators have high sentiment, extrapolation indicates that the recent returns in the past were high and the market is undergoing an upward correction. Therefore, high sentiment implies good future returns along this upward path. Conversely, if extrapolators have low sentiment, extrapolation indicates a downward correction, and fundamental investors will continue shorting risky asset until asset price reaches its fundamental value. Low sentiment predicts negative returns along this path. Together, conditional on the low wealth level of extrapolators, investor sentiment positively predicts future market returns.

Figure 3 provides intuitive explanations for the positive predictive pattern of investor sentiment when the wealth level of extrapolators is low. If investor sentiment is low, extrapolation indicate that the asset price is undergoing a upward correction. Therefore, the only possible price dynamics is the one that goes up but is below the fundamental value—the wealth level of extrapolators is too low to push asset prices upwards from the fundamental value. Conversely, if investor sentiment is low, then the only possible situation is that the asset price is corrected downwards from overvaluations—the low market impact of extrapolators is not able to push asset prices downwards from the fundamental value.

[Place Figure 3 about here]

IV. Model Implications

In this section, I provide analyses for the behavioral model based on simulations. Specifically, I start by checking the belief pattern of extrapolators to see whether it captures the investor sentiment dynamics in the survey. Then I test whether there is a strong connection between investor sentiment and market mispricing, and test the key model implications on the return predictability based on investor sentiment. I also use my model to shed some light on the extant asset pricing patterns in the empirical literature.

A. Extrapolators' Beliefs

One important prerequisite to investigate the impact of investor sentiment is to understand the dynamics of investor sentiment. In this subsection, I provide justifications for the belief structure in my behavioral model based on model simulations. Specifically, I first compare investor sentiment based on model simulations with that in the Gallup survey. Second, I regress the model-implied investor sentiment on either past twelve month returns or the current log price to dividend ratio, and check whether my model embodies extrapolative expectations. Third, I formally test how much weight extrapolators put on the past realized returns and compare my results to the literature. Last, I test whether the model-implied investor sentiment reverts to its mean.

To simulate my model, I back out a sequence of shocks based on the monthly real dividend data of the S&P 500 index starting from June 1996 to December 2011. This range is consistent with the Gallup series in Greenwood and Shleifer (2014) and therefore can facilitate my comparison between the simulated sentiment series and the survey data. Moreover, I also use these implied shocks for my analyses of the behavioral model, so that I can compare two models within the same background.

Specifically, in order to get the series of shocks, I take the log on the dividend process and then use Ito's lemma to get

$$d\ln D_t = (g_D - \frac{1}{2}\sigma_D^2)dt + \sigma_D d\omega_t. \tag{30}$$

Then, I can discretize the equation and back out the shocks using the following formula:

$$\epsilon_t = (\ln D_{t+1} - \ln D_t - (g_D - \frac{1}{2}\sigma_D^2)\Delta_t)/(\sigma_D\sqrt{\Delta_t}). \tag{31}$$

Since the real dividend data is at a monthly frequency, I set Δ_t equals to 1/12. In addition, I set g_D as 1.5% and σ_D as 10%, which are both commonly accepted magnitudes in the asset pricing literature. I take a neutral stand and set the initial sentiment level to be $S_0 = 0$. I also set the initial wealth to dividend ratio x_0 to be 1. Since I have numerically solved the equilibrium, I can easily get the simulated sequence for investor sentiment S_t, price process P_t, the wealth-dividend ratio process x_t as well as $l(S_t, x_t)$, $g_{P,t}(S_t, x_t)$ and $\sigma_P(S_t, x_t)$.

A.1. Model-implied Investor Sentiment

Since both my model simulation and Gallup survey are driven by the same underlying shocks to the economy, I can directly compare these two sequences. I plot both series in Figure 6. From this figure, it is clear that the simulated investor sentiment based on the behavioral model is highly synchronized with the Gallup survey series, and captures the large part of anecdotal fluctuations of investor sentiment in the market. For example, consistent with the Gallup survey, the investor sentiment based on the behavioral model also rises to its peak right before the 2007 financial crisis and drops significantly during the afterward recessions.

[Place Figure 6 about here]

A.2. Extrapolative Belief Structures

Next, I zoom in the extrapolation pattern in my behavioral model. Greenwood and Shleifer (2014) documents that investor expectations about future returns heavily depend on the current price level and past realized returns. Specifically, I regress the perceived expectations of future returns on either the current log price-dividend ratio or the past twelve-month accumulative raw returns, based on the simulated series. I report the regression coefficients, their t-statistics and their R-squared in Table VII. For robustness check, I report results based on two different types of expectation measures: the expectations of future return growth rate $dP_t/(P_t dt)$ and the expectations of future return growth rate with dividend yield $dP_t/(P_t dt) + l^{-1}$.

Table VII indicates the investor belief pattern in my model matches the extrapolation pattern documented in Greenwood and Shleifer (2014). Among others, both in my model and in the survey, subjective expectations on future returns are positively related to the current log price-dividend ratio and the past twelve-month returns. Moreover, the regression coefficients and t-statistics are also close to the regression results based on the Gallup survey. For instance, compared to the regression coefficient of 9.12% and t-statistics of 8.81 from regressions based on Gallup survey, my simulation-based regression yields coefficient of 2.3% and t-statistics of 2.09.

[Place Table VII about here]

A.3. Extrapolators' Memory Span

Another important dimension of investor belief structure is its memory span, which measures how much weight extrapolators put on the recent returns. In my model, extrapolators' memory span is controlled by the magnitude of belief parameters χ and λ, which determines how fast extrapolators update their beliefs. To formally test extrapolators' memory span, I run the nonlinear regression following Greenwood and Shleifer (2014):

$$\begin{aligned} \text{Expectation}_t &= a + b\Sigma_{s=0}^{\infty}\omega_s R_{t-(s+1)\Delta t, t-s\Delta t}, \\ \omega_s &= \frac{e^{-\psi s \Delta_t}}{\Sigma_{k=0}^{n} e^{-\psi k \Delta_t}}, \end{aligned} \tag{32}$$

In Table VIII, I report the regression coefficient a, the intercept b, the adjusted-R^2, and more importantly, the estimated memory span parameter ψ. My simulation is at monthly frequency and therefore I set $\Delta = 1/12$. I impose $n = 600$, which means extrapolators use returns within the past 50 years to form their beliefs. Table VIII show that the estimated ψ is 0.51. This means a monthly returns three years ago is weighted only 25% as much as the most recent returns. As a comparison, Barberis et al. (2015) obtain an estimator of 0.44 for ψ. This consistency in turn helps justify the belief parameter choice for χ and λ.

It is worth pointing out that there is no consensus on investors' memory spans in the behavioral finance literature. Some studies, including Greenwood and Shleifer (2014) and Kuchler and Zafar (2016), document that investors have short memories and only use recent information in the past few years to form their beliefs. By contrast, other studies such as Malmendier and Nagel (2011), Malmendier and Nagel (2013) and Vanasco, Malmendier, and Pouzo (2015) argue that investors form their beliefs based on their life experience—events in the distant past might still have a significant impact in the belief formation process. Explaining this discrepancy is beyond the scope of this paper, but correctly interpreting investors' memory span would help understand the investor beliefs and the asset price dynamics.

[Place Table VIII about here]

A.4. Model-implied Mean-reversion Property of Investor Sentiment

To complete my justification on my belief structure, I check whether investor sentiment in my model reverts to its mean. Consistent with Observation 2, I run the regression in equation (3), but replace survey measures with the model-implied investor sentiment.

[Place Table IX about here]

In table IX , I report the regression coefficients, t-statistics and the adjusted-R^2. All t-statistics are adjusted with Newey-West correction. In addition, for robustness check, I also report results of forecasting horizons from 1 to 6 quarters. Table IX shows that the coefficient of sentiment variable is uniformly negative and statistically significant, which indicates that investor sentiment in my model reverts to its mean.

With the extrapolation pattern in my behavioral model, now I investigate the time-varying impact of investor sentiment.

B. Impact of Investor Sentiment

B.1. The Interaction Effect in the Model

In this section, I test whether the interaction effect in Observation 3 exists in my model. The market clearing condition (20) has already provides some clues: sentiment-driven risk exposure α_t and market power W_t together determines the equilibrium asset price. However, to provide a direct comparison, I rely on the model-simulated time series to test the interaction effect in my model.

Following the analysis on Observation 3, I run the regression:

$$R^e_{t+N} = a + b\text{Sent}_t + cW_t/D_t + d\text{Sent}_t \times W_t/D_t + \epsilon_t, \tag{33}$$

where R^e_{t+N} represents the future excess returns over the next N months, Sent_t represents model-simulated investor sentiment, and W_t/D_t proxies for the wealth to dividend ratio. These two variables are also the underlying state variables in my model.

[Place Table X about here]

Table X reports the regression results based on the model-simulated data. The coefficient for our interaction term is statistically significant. Therefore, the interaction effect is confirmed in my theoretical model.

B.2. Time-varying Impact of Investor Sentiment

Based on my behavioral model, investor sentiment has time-varying impacts on the market. To formally test this model prediction, I empirically test the predictive power of investor sentiment conditional on different market impacts of extrapolators. In section IV.B.1, I already connect the market impact of extrapolators with mispricing. However, the predictive direction of investor sentiment still remains ambiguous.

To provide a more detailed analysis, I present the predictive coefficient of investor sentiment $b_t = [b + d \times W_t/D_t]$ for future 12-month returns in Table XI. Specifically, I follow Aiken et al. (1991) and present the predictive pattern of investor sentiment conditional on different wealth levels. In the remaining analysis, I define the high wealth level as two standard deviations above the mean of the wealth to dividend ratio, and the low wealth level as two standard deviations below the mean of the wealth to dividend ratio.

[Place Table XI about here]

The results coincide with my theoretical model predictions. When the wealth level of extrapolators is high, one unit increase in Gallup-measured investor sentiment is followed by a significant 16.2% decrease in the future 12-month returns. Replace Gallup survey with Psentiment, the conditional predictive pattern is even stronger: one unit in Psentiment, in general, leads to a tremendous return decline of 26.7% in the next 12 month. If extrapolators have high market power, then they are driving the market and investor sentiment negatively predicts future returns because it reverts to its mean in the future. Such negative predictive pattern is hardly consistent with rational asset pricing models.

Moreover, when the wealth level of extrapolators is low, investor sentiment has a strong positive predictive power. With Gallup survey and Psentiment, the coefficients of the conditional coefficients b_t are 0.359 and 0.479 respectively. At first glance, positive coefficient might be inconsistent with the irrational story of investor sentiment. However, my model indicates they are consistent. The key observation from my model is that, when extrapolators' wealth level is low, the main market power comes from fundamental investors. In this situation, high sentiment indicates the market is going through an upward market correction, leading to positive market returns in the future. Similarly, low sentiment implies that fundamental investors are correcting overvalued asset prices downwards, causing negative market returns.

The regression results based on model simulations in Table XII also confirms the conditional predictive pattern of investor sentiment in the model – they are very close to the predictive pattern in the empirical data.

It is worth pointing out that, conditional on the wealth level of extrapolators, investor sentiment has significant predictive power over future market returns. By contrast, without considering the market impact of extrapolation, investor sentiment does not have significant predictive power over future market returns, as shown in Table III.

C. Comparison between Interaction Effect and Degree of Extrapolation

Both my theoretical and empirical results support that the predictive power of investor sentiment on future returns is time-varying. A similar pattern is recently recorded in Cassella and Gulen (2015). In their paper, they define the degree of extrapolation (DOX) as the relative weight extrapolators place on recent-versus-distant past returns when forming their sentiment on future stock market returns. Their key empirical findings are that DOX is time-varying and that predictability of log price-dividend ratio strongly depends on the DOX. Higher (lower) log price to dividend ratio indicates assets are undervalued (overvalued). When DOX is higher and extrapolators rely more on recent returns, few recent returns will change both their sentiment and irrational demand dramatically, which induces a faster correction of mispricing. These two sets of conditional predictive patterns seem to be fundamentally similar and connected to each other, since both answers one critical question in the asset pricing theory: when will a mispriced asset experience a correction?

[Place Figure 7 about here]

Specifically, their explanation to this question relies on the aggregate belief transition pattern of extrapolators (DOX), which could either depend on the time-varying participation rate of extrapolators or the time-varying return extrapolation pattern at the individual investor level. With higher DOX, investors easily change their sentiment based on recent returns, which leads to a faster mispricing correction speed and stronger predictive pattern of log price to dividend ratio.

As a comparison, the interaction effect based on my results proves the importance of the time-varying impact of of investor sentiment. When their wealth level is high, investor sentiment leads to higher degree of asset mispricing, which leads to a strong mispricing correction in the future and a significant predictive power of investor sentiment. Two explanations should have overlapped common underlying mechanism.

Given this similarity, a natural hypothesis is that the DOX in their paper might be very correlated with the interaction effect in my model. To check it, I plot both DOX and the interaction effect in Figure 7. As we can see from the figure, two measurements are indeed highly synchronous. For instance, between 1996 and 2000 and before the Internet bubble bursts, the DOX measurements rise from 0.35 to 0.9. Within the same sample period, the market impact in my model also increases. Therefore, my model provides a formal justification for the time-varying DOX: variations in DOX is largely due to the time-varying impact of extrapolators in the market.

Despite the similarities, there are some new predictions using the interaction term in my model. For example, as I have shown, the time-varying impact of extrapolators implies a novel predictive power of *investor sentiment* on future returns.

D. Household Leverage and Future Consumption Growth

My model also sheds lights on the recent findings in the household literature. Mian and Sufi (2009) document that household leverage ratio before the Great Depression is a strong and negative predictor of future consumption growth rate. Moreover, they document that the increase in household leverage is largely due to the biased expectations, and belief mistakes lead to a decrease in future income and declines in consumption growth rate.

My model provides a formal justification for this mechanism. In my model, extrapolators form biased beliefs based on past returns, consequently, they become overly optimistic and take excessive leverage. However, their high expectations will not continue for long and consequently, the wealth level of extrapolators in the future will decline. Since the optimal consumption rule is linearly depending on the wealth level, their future consumption rate will also decline as the result.

My model simulation results confirm this. In table XIII, I report the predictive regression of future consumption growth using current leverage ratio as the predictor. The predictive pattern is significantly negative, and remains stable when forecasting horizons span from 1 to 6 quarters.

E. Additional Model Implications

E.1. Bubbles

Extrapolative beliefs help generate excessive optimism and pessimism in the market, consistent with the anecdotal descriptions of market boom and bust. Specifically, episodes of good returns lead extrapolators to become overly optimistic about the future expected returns, consistent with the high sentiment during booms measured in the survey. Conversely, a sequence of low returns will significantly disappoint extrapolators, leading to a drop in investor sentiment during recessions.

Moreover, what distinguishes this model from others in explaining the bubble phenomena is the role of the wealth dynamics during the bubble and crash episodes. In my model, investor sentiment has large impacts on the dynamics of the wealth process, especially during bubbles when the wealth level of extrapolators is high. High sentiment during bubbles induces investors to lever up, which pushes up the total dollar demand and leads to overvaluations. However, investor sentiment could not last long since extrapolators get disappointed by future shocks, and future dollar demand for the risky asset goes down. The decrease in risky asset price not only makes investors pessimistic but also reduces the overall market impact of extrapolators, which makes fundamental investors easier to correct mispricing and leads to the market crash.

[Place Figure 8 about here]

E.2. Countercyclical Sharpe ratios and Negative Risk Premium

My model with extrapolation naturally generates a procyclical perceived Sharpe ratio, since investor beliefs are mainly driven by recent realized returns in the past: during market booms, investors become optimistic about future returns and the perceived Sharpe ratio increases. Conversely, in the recessions, investors become overly pessimistic and perceive low Sharpe ratio due to low past returns. This procyclicality resonates with the finding in Amromin and Sharpe (2013) where they document a procyclical Sharpe ratios using data from Michigan Surveys of Consumer Attitudes.

Compared to rational expectation models, the behavioral model in my paper has two distinct probability measures. While Sharpe ratio is procyclical under the *perceived* probability measure, my model generates a *countercyclical* Sharpe ratio under the *objective* probability measure. This countercyclicality closely relates to the fact that investor sentiment reverts: with high level of investor sentiment, extrapolators become overly optimistic and perceive high Sharpe ratio, but the Sharpe ratio under objective probability measure is low in the future. This countercyclical Sharpe ratio is consistent with the empirical pattern documented in Lettau and Ludvigson (2010).

E.3. Debt Accumulation and Procyclical Leverage

Many studies document the strong cyclicality of debt accumulation. For instance, Reinhart and Rogoff (2009) document that household debt accumulation speeds up during market booms and often leads to severe financial crashes. The extrapolation in my model capture this dynamic pattern for debt. During market booms, extrapolators become overly optimistic by extrapolating the past returns and consequently they take lever up to buy the risky asset. Conversely, during recessions, extrapolators become overly pessimistic and therefore have low leverage level. In addition, consistent with many other models such as He and Krishnamurthy (2008), this model also captures the procyclical leverage. However, different from previous studies, the procyclicality in my model arises from extrapolation.

V. The Role of Extrapolation: A Rational Benchmark Model

To correctly evaluate the role of extrapolation, in this section, I impose extrapolators to have fully correct beliefs by setting $\theta = 0$, and compare the model implications with previous behavioral model settings. Such comparison implies that extrapolation is important for understanding investor beliefs and asset price dynamics.

A. The Economy

In this rational benchmark model, I follow previous setting and consider a continuous-time economy with two types of assets: one risk-free asset and one risky asset. The underlying dividend process D_t for the risky asset still follows a geometric Brownian motion:

$$dD_t/D_t = g_D dt + \sigma_D d\omega_t. \tag{34}$$

The dividend process is driven by ω_t, a one-dimensional Weiner process under the true probability measure observed by outside econometricians. The equilibrium price P_t now follows

$$dP_t/P_t = \bar{g}_{P,t} dt + \bar{\sigma}_{P,t} d\omega_t. \tag{35}$$

The growth rate $g_{P,t}$ and volatility term $\sigma_{P,t}$ are both endogenously determined in the equilibrium.

B. Investors

There are two types of investors in the rational benchmark model: extrapolators and fundamental investors. Different from the case of behavioral model, extrapolators have correct beliefs about the asset dynamics and optimize their portfolio choice under the objective probability measure accordingly. Fundamental investors, on the other end, still serves as the counteracting forces in the market and trade aggressively whenever asset prices deviate from fundamental values. Similarly, I assume that fundamental investors make up a fraction of $1-\mu$ and extrapolators make up a fraction of μ.

B.1. Extrapolators

As in the behavioral model, I introduce logarithmic utilities for extrapolators and assume that each of them, indexed by j, is infinitesimal and therefore acts as price-taker in the market. They solve the following optimization problem:

$$\max_{\{C^j_{t+s}\}_{s\geq 0}} \mathbb{E}_t[\int_t^\infty e^{-\rho s} \ln C^j_s ds], \tag{36}$$

and subject to budget constraint

$$dW^j_t = -C^j_t dt + rW^j_t dt + \alpha^j_t W^j_t [\bar{g}_{P,t} dt + \frac{D_t}{P_t} dt - rdt + \bar{\sigma}_{P,t} d\omega_t],$$

Most of the properties of the optimal strategies in the behavioral model – that the optimal risky position depends on instantaneous equity premium and volatility, that the investment decision is myopic and ignores the hedging demand – carry over to the rational benchmark case. However, one key difference is that now extrapolators make investment decisions under the true probability measure.

Following the standard Merton's approach, we get the optimal consumption and portfolio rule for extrapolators:

PROPOSITION 3: *In the rational benchmark economy, the extrapolators with the objective function in equation (36) and budget constraint in equation (37) will optimally consume and invest at time* *t according to the following strategies:*

$$C_t^j = \rho W_t^j \tag{37}$$

and

$$\alpha_t^j = \frac{\bar{g}_{P,t}(x_t) + \bar{l}^{-1}(x_t) - r}{\bar{\sigma}_{P,t}^2(x_t)}. \tag{38}$$

where the price to dividend ratio $\bar{l} \equiv \frac{P_t}{D_t}$ depends on the wealth to dividend ratio $x_t \equiv \frac{W_t}{D_t}$. In other words, the equilibrium depends on state variable x_t.

Proof: See Appendix B.

C. Equilibrium

Definition of Equilibrium In this rational benchmark model, the equilibrium satisfies the following conditions:

(i) Rational investors follow the optimal consumption and portfolio choice described in proposition (3).

(ii) Fundamental investors follow their trading strategy defined in equation (19).

(iii) The following market clearing condition holds:

$$\mu \alpha W_t + (1-\mu) Q_t = P_t \tag{39}$$

For the rational benchmark model, I still focus on the symmetric equilibrium where each extrapolator is endowed with the same level of wealth and follows identical strategies. The equilibrium depends both on the wealth to dividend ratio x_t and the latent state variable S_t. Therefore, I rely on solving a partial differential equation to obtain the solution for the fixed-point problem. As verified later, the price to dividend ratio is monotonically increasing in both x_t and S_t.

PROPOSITION 4: *In the symmetric equilibrium, the price-dividend ratio $\bar{l}$ is a function of the* *current states x_t, and satisfies the following partial differential equation:*

$$\frac{c_0\bar{l} - c_1}{x_t} = \frac{\bar{g}_{P,t} + \bar{l}^{-1} - r}{\bar{\sigma}_{P,t}^2}, \tag{40}$$

where $\sigma_{P,t}$ is also a function of x_t and satisfies

$$\bar{\sigma}_{P,t} = \sigma_D \frac{1 - \frac{\bar{l}_x}{\bar{l}} x_t}{1 - \frac{\bar{l}_x}{\bar{l}} x_t \alpha_t} \tag{41}$$

and $l(x_t)$ satisfies the following boundary conditions

$$\lim_{x_t \to 0} \bar{l} = \frac{c_1}{c_0}, \tag{42}$$

$$\lim_{x_t \to 0} = \lim_{x \to 0} \frac{g_D + \frac{c_1}{c_0} - r}{\sigma_D^2}, \tag{43}$$

and

$$\lim_{x_t \to \infty} \bar{l} = (r - \bar{g}_{P,t})^{-1}, \tag{44}$$

where $c_0 = \frac{k+1-\mu}{k\mu}$ and $c_1 = \frac{1-\mu}{k\mu(r-g_D)}$ are both constants.

In addition, the dynamics of x_t follows

$$dx_t = g_{x,t}(x_t)dt + \sigma_{x,t}d\omega_t, \tag{45}$$

where

$$g_{x,t} = x_t(r - \rho - g_D + \sigma_D^2) + (c_0 l - c_1)(\bar{g}_{P,t} + l^{-1} - r - \sigma_D \bar{\sigma}_{P,t}) \tag{46}$$

$$\sigma_{x,t} = (c_0 l - c_1)\bar{\sigma}_{P,t} - x_t \sigma_D \tag{47}$$

Proof: See Appendix B.

The volatility pattern in equation (41) essentially reflects the wealth effect and the portfolio rebalancing effect, as documented in previous studies (Xiong (2001) and Jin (2015)). When α_t is greater than one, extrapolators borrow money at the risk-free rate r and lever up their positions in the risky asset. Once positive dividend shocks arrive, the increase in the risky asset leads to an increase in their wealth level and hence their absolute risk aversion; at the same time, the leverage ratio of extrapolators is further pushed down, which induce them to buy more risky assets. This wealth effect amplifies the initial dividend shocks, which makes $\bar{\sigma}_{P,t}$ larger than σ_D. Conversely, when α_t is less than one, the portfolio rebalancing effect dominates: once positive dividend shocks arrive, the increase in the risky asset leads to an increase in their wealth level and hence their absolute risk aversion; at the same time, the leverage ratio of extrapolators is further pushed up, which induce them to delever. Together, the portfolio rebalancing effect dampens the initial shocks and make $\bar{\sigma}_{P,t}$ less than σ_D.

In solving the ordinary equation in proposition (4), I still rely on numerical methods. Similarly, I apply the following monotonic transformation

$$z_t = \frac{x_t - \xi}{x_t + \xi}, \tag{48}$$

where ξ is a positive constant.

D. Calibrated Model for the Benchmark Case

In this section, I solve the rational benchmark model using the projection method. I use the following parameter values: $r = 4\%$, $g_D = 1.5\%$, $\sigma_D = 10\%$, $\rho = 2\%$, $k = 0.5$ and $\mu = 0.5$. The selected asset and utility parameters are reported in the upper two panels in Table VI.

[Place Table VI about here]

[Place Figure 4 about here]

D.1. Price-Dividend Ratio: Benchmark Case

With the above parameters, I report the model solution in Figure 4. In the upper panel, I report the price-dividend ratio $\bar{l}(x_t)$ as a function of the wealth-to-dividend ratio. As we can see, $\bar{l}(x_t)$ is an monotonically increasing function of x_t. When x_t equals zero, the risky asset is hold by the fundamental investors and the risky asset price is at its fundamental value, which is relatively low. Increase in the wealth level of extrapolators helps push up the total dollar amount of the risky asset, which in turn increases the price-to-dividend ratio $\bar{l}(x_t)$. With the given parameters, the price to dividend ratio lies between 20 to 40, which is fairly consistent with the empirical moments in the literature.[18]

D.2. Optimal Portfolio Weight: Benchmark Case

The portfolio weight α_t for extrapolators is monotonically decreasing in z_t. Recall that the price to dividend ratio positively depends on the wealth to dividend ratio. Therefore, as z_t increases, the dividend yield naturally goes down, which makes the investment set less attractive to the extrapolators. Consequently, extrapolators want to decrease their position in the risky asset, reducing the optimal portfolio α_t.

D.3. Volatility: Benchmark Case

I also report the volatility $\bar{\sigma}_{P,t}(x_t)$ in the equilibrium as a function of the wealth to dividend ratio. As discussed before, the reverse-S shaped curve is closely related to extrapolators' portfolio choice α_t. When the transformed wealth-to-dividend ratio z_t is slightly below 1, the wealth fraction they invested in the risky asset grows from zero to slightly above. When α_t is positive but small, faced with exogenous shocks, investors will rebalance their portfolio and dampen the overall volatility. As a result, equilibrium $\bar{\sigma}_{P,t}$ is less than the magnitude of fundamental shocks σ_D and it can be as low as 9.55%. As z_t decreases and the portfolio weight α_t gradually becomes greater than one, a negative dividend shock will cause their leverage ratio to increase and lead extrapolators to reduce their risky asset position, which helps amplify the initial shocks and pushes up volatility $\bar{\sigma}_{P,t}$. When z_t goes down to -1, extrapolators lose their impact and fundamental investors gradually dominate the market, which makes the volatility gradually converge to σ_D.

E. Extrapolators' Beliefs

Extrapolators in the rational benchmark model have fully correct beliefs about future risky asset returns. As a result, their belief pattern is *not* consistent with the observed dynamics for investor sentiment. To provide a formal justification, I still focus on the model simulation results.

[18]Beeler, Campbell, et al. (2012) report the average of log(PD) ratio of around 28.8.

[Place Table XIV about here]

First, I follow Greenwood and Shleifer (2014) and run the following regression:

$$\mathbb{E}[R_{t+12}] = a_0 + a_1 X_t + \epsilon_t \tag{49}$$

where $\mathbb{E}[R_{t+12}]$ represents the expectation of future twelve-month returns under the true probability measure, X_t represent either past accumulative 12-month returns or the current log price to dividend ratio.

I report the regression results in Table XIV. The belief pattern for extrapolators in the rational benchmark model is not consistent with the extrapolation pattern in the survey: when the current log price to dividend ratio is high, extrapolators with correct beliefs would anticipate that the market is overvalued and then anticipate a decline in the risky asset price.

F. The Interaction effect

[Place Table XV about here]

Moreover, the rational benchmark model does not capture the strong interaction effect between investor sentiment and the wealth level of extrapolators in determining the mispricing. Specifically, I test the Observation 3 by running the same regression in equation (4) using the simulated series from the rational benchmark model. The regression results are reported in Table XV. The coefficients for each of the regressors are all statistically significant but positive because extrapolators always hold correct beliefs.

G. Household Leverage and Future Consumption Growth

The absence of extrapolation also drives away the negative association between the household leverage and the future consumption growth. In the rational model, households have fully correct beliefs, therefore, they increase their exposure to the risky asset when future returns are high. This leads to a positive correlation between the leverage ratio and future consumption growth rate, which is inconsistent with the pattern documented in Mian and Sufi (2009). As shown in Table XVI, when regressing future consumption growth rate on current leverage ratio, the coefficient α_t

is significantly positive.

In summary, with fully correct beliefs, rational models miss most of the important model implications in the behavioral model. Therefore, extrapolation, or equivalently investor beliefs, plays a key role in explaining a set of empirical patterns. The excessive optimism and pessimism induced by extrapolation not only help understand the anecdotal fluctuations in investor sentiment but also helps explain the asset mispricings and consequently the predictability of returns in the aggregate stock market. Moreover, it helps shed light on the household leverage-taking behaviors and corresponding real consequences to the economy. All these facts point to the importance of belief-based investor sentiment in understanding both asset pricing facts and real economic activities.

VI. Concluding Remarks

In this paper, I have developed a model that focuses on investor sentiment and analyzed its time-varying impact on the market. In my model, there are two types of investors: fundamental investors who trade as aggressive arbitragers and extrapolators who form investor sentiment by over-extrapolating past realized returns. The model implies dynamic connections between investor sentiment and market mispricing through the market impact of extrapolators. When the wealth level of extrapolators is high and therefore have a large impact on the equilibrium, investor sentiment directly causes the market mispricing and negatively predicts future market returns due to the fact that investor sentiment reverts quickly to its mean. Conversely, if the wealth level of extrapolators is low – the situation in which fundamental investors dominate the market and extrapolators have trivial impacts on the market – investor sentiment positively predicts future market returns since it reflects an undergoing market correction.

There are at least two avenues for future work. First, a fully fledged model that simultaneously incorporates extrapolators, fundamental investors and rational investors are in demand. Although intuition tells me that the existence of rational investors will only reinforce my current results of the behavioral model due to their "ride on the bubble" motives, a formal model will help verify my conjecture. Second, my model implies a strong but general connection between investor sentiment and mispricing by the time-varying impact of extrapolators, and I find empirically support using the stock market as an example. However, the connection implied in my model should be a general pattern that applies to a broader set of asset classes. Empirically testing such connections in other asset classes will deepen our understanding of the impact of investor sentiment.

Appendix A. Micro-foundations for Fundamental Investors

To keep the decision problem of fundamental investors simple, I model the fundamental investors as an overlapping generation (OLG) of agents. For the sake of clarity in explaining the OLG timeline in a continuous time setting, I index time as t, $t+\Delta t$, $t+2\Delta t$ and so on. Each fundamental investor in generation t inherits wealth from the last generation and lives between period t, and $t+\Delta t$. For simplicity, fundamental investors are assumed to have homogeneous wealth levels of W_t^f at the beginning of period t and adjust their portfolio to maximize the exponential utility with respect to their bequest to the next generation, $W_{t+\Delta t}^f$.

One crucial assumption here, however, is fundamental investors can liquidate their risky asset at the fundamental value $P_{F,t} \equiv \frac{D_t}{r-g_D}$ at $t+\Delta t$.[19] There are multiple ways to justify this assumption. One possible situation is that at the end of each period, fundamental investors can trade with mutual funds who target on fundamental values of assets with a price of $P_{F,t}$.

The excess return of holding the risky asset for fundamental investors is therefore $\frac{P_{F,t}-P_t}{P_t}$, Moreover, their trade counterparts also suffer from exogenous liquidity shocks, $\tilde{\epsilon} \sim N(r-1, \frac{\sigma_\epsilon^2}{P_t})$.[20] The liquidity shock also affects the resale returns of the risky asset price – with a realized shock of ϵ_t, holding risky asset only yields an excess return of $\frac{P_{F,t}-P_t}{P_t} - \epsilon_t$.

The timeline for fundamental investors is as follows. At the beginning of period t, fundamental investors receive bequest amount of W_t^f and observe the risky asset price, P_t, as well as the fundamental value, $P_{F,t}$. They select their risky asset position to maximize:

$$U(W_{t+\Delta t}^f) = -exp(-\gamma_h W_{t+\Delta t}^f). \tag{A1}$$

with budget constraint

$$W_{t+\Delta t}^f = W_t^f r + \alpha_t^f W_t^f (1 + \frac{P_{t+\Delta t} - P_t}{P_t} - \epsilon_t - r) \tag{A2}$$

where γ_h represents the risk aversion coefficient of fundamental investors and α_t^f is the total risky

[19] The fundamental value of the risky asset follows the Gordon growth formula.

[20] Such shocks can be motivated by liquidity constraints. Intuitively, the mean value of liquidity shock is positively correlated with interest rate r: higher interest rate elevate the liquidity shock. Also, the variance of ϵ is inversely correlated to the price level of the risky asset, with higher risky asset price associated with lower liquidity risk.

asset demand of fundamental investors. The first order condition gives the optimal portfolio choice:

$$\alpha_t^f = \frac{P_{F,t} - P_t}{\gamma_h \sigma_\epsilon^2 W_t^f} \tag{A3}$$

Therefore, the per-capita total dollar demand of fundamental investors is

$$Q_t \equiv \alpha_t^f W_t^f = \frac{P_{F,t} - P_t}{\gamma_h \sigma_\epsilon^2} \equiv \frac{P_{F,t} - P_t}{k}, \tag{A4}$$

where k is a constant.

Appendix B. Rational Benchmark Model

Following the standard Merton method, we have the optimal portfolio for the extrapolators with fully correct beliefs:

$$\alpha_t = \frac{\bar{g}_{P,t} + l^{-1} - r}{\bar{\sigma}^2_{P,t}} \tag{B1}$$

Using Ito's lemma on both sides of $x_t \equiv \frac{W_t}{D_t}$:

$$\begin{aligned} dx_t = d(W_t/D_t) =& x(dW_t/W_t - dD_t/D_t + (dD/D)^2 - (dW/W)(dD/D)) \\ =& x(r - \rho + \alpha_t(\bar{g}_{P,t} + l^{-1} - r) - g_D + \sigma_D^2 - \alpha_t \sigma_D \bar{\sigma}_{P,t})dt + x(\alpha_t \bar{\sigma}_{P,t} - \sigma_D)d\omega_t \\ \equiv& g_x dt + \sigma_x d\omega_t \end{aligned} \tag{B2}$$

Combined with the geometric form of the dividend process and the logarithmic form utility function, I conjecture that the underlying state of the economy is $x_t \equiv \frac{W_t}{D_t}$, the wealth to dividend ratio. In other words, all equilibrium quantities can be expressed as a function of x_t. For example, the price to dividend can be denoted as $\bar{l}(x_t) = \frac{P_t}{D_t}$.

In addition, for notational convenience, I also denote the dynamics of x_t as:

$$dx_t = g_{x,t}(x_t)dt + \sigma_{x,t}d\omega_t. \tag{B3}$$

Then I apply Ito's lemma on both sides of $l = P/D$ and get:

$$dl = l_x dx + \frac{1}{2} l_{xx} \sigma_x^2 dt = (l_x g_x + \frac{1}{2} l_{xx} \sigma_x^2)dt + l_x \sigma_x d\omega_t \tag{B4}$$

$$\begin{aligned} d(P_t/D_t) =& dP_t/P_t - dD_t/D_t + (dD_t/D_t)^2 - (dD_t/D_t)(dP_t/dP_t) \\ =& l(\bar{g}_{P,t} - g_D + \sigma_D^2 - \sigma_D \bar{\sigma}_{P,t})dt + l(\bar{\sigma}_{P,t} - \sigma_D)d\omega_t \end{aligned} \tag{B5}$$

By matching terms:

$$l_x g_x + \frac{1}{2} l_{xx} \sigma_x^2 = l(\bar{g}_{P,t} - g_D + \sigma_D^2 - \sigma_D \bar{\sigma}_{P,t}) \tag{B6}$$

$$l_x \sigma_x = l(\bar{\sigma}_{P,t} - \sigma_D) \tag{B7}$$

Then by combining equation (B2) and (B6), I can solve for $\bar{g}_{P,t}$:

$$\bar{g}_{P,t} = \frac{l_x x(r - \rho - g_D + \sigma_D^2) + l_x(c_0 l - c_1)(l^{-1} - r - \sigma_D \bar{\sigma}_{P,t}) + \frac{1}{2} l_{xx} \sigma_x^2 + l(g_D - \sigma_D^2 + \sigma_D \bar{\sigma}_{P,t})}{l - l_x(c_0 l - c_1)} \tag{B8}$$

Then I can get

$$g_x = x_t(r - \rho - g_D + \sigma_D^2) + (c_0 l - c_1)(\bar{g}_{P,t} + l^{-1} - r - \sigma_D \bar{\sigma}_{P,t}) \tag{B9}$$

and solve for

$$\bar{\sigma}_{P,t} = \sigma_D + \frac{l_x}{l} x(\alpha_t \bar{\sigma}_{P,t} - \sigma_D) \tag{B10}$$

With $x_t \alpha_t = c_0 l - c_1$, I have:

$$\bar{\sigma}_{P,t} = \sigma_D \frac{1 - \frac{l_x}{l} x_t}{1 - \frac{l_x}{l}(c_0 l - c_1)} \tag{B11}$$

$$\sigma_x = (c_0 l - c_1)\bar{\sigma}_{P,t} - x_t \sigma_D$$

Combined with the market clearing conditions and the Chebyshev polynomials (for more details, see appendix C), I can numerically solve this model.

Appendix C. Behavioral Model

In the behavioral model, the time-varying investment set is driven by two state variables: x_t and S_t. Therefore, using the standard argument in Merton (1971), I define the extrapolators' value function as

$$J(W_t, S_t, x_t) \equiv \max_{\{C_{t+s}\}_{s\geq 0}} \mathbb{E}_t^e[\int_t^\infty e^{-\rho s} \ln C_s ds] \tag{C1}$$

For the logarithmic form utility function, I guess the corresponding value function has the form

$$J(W_t, S_t, x_t) = 1/\rho \ln(W_t) + j(S_t, x_t), \tag{C2}$$

Under extrapolators' subjective beliefs, after omitting the subscripts, the Hamilton-Jacobian-Bellman (HJB) equation follows

$$\begin{aligned} \rho J(W,S,x) =& ln(W) + \rho j(S,x) \\ =& \max_{C,\alpha} \big[\ln C + J_W W[-C/W dt + rdt + \alpha[\hat{g}_{P,t} + l^{-1} - r]dt] + 1/2 J_{WW}\alpha^2 W^2 \sigma_{P,t}^2 dt \\ & + J_S \mu_S dt + 1/2 J_{SS}\sigma_S^2 dt + J_x g_x dt + 1/2 J_{xx}\sigma_x^2 dt + J_{Sx}\sigma_S\sigma_x dt \big], \end{aligned} \tag{C3}$$

where J_{WS} and J_{Wx} all equal to zero and are omitted.

By the FOC $w.r.t$ C, I have

$$1/C_t - J_W = 0,$$

which implies

$$C_t = \rho W_t. \tag{C4}$$

Substitute equation (C4) into the HJB equation (C3), I have the following optimization problem with respect to α_t:

$$\begin{aligned} &\max_{\alpha} \big[\ln \rho + J_W W[-\rho dt + rdt + \alpha(\hat{g}_{P,t} + l^{-1} - r)dt] + 1/2 J_{WW}\alpha^2 W^2 \sigma_{P,t}^2 dt \\ &+ J_S \mu_S dt + 1/2 J_{SS}\sigma_S^2 dt + J_x g_x dt + 1/2 J_{xx}\sigma_x^2 dt + J_{Sx}\sigma_S\sigma_x dt \big], \end{aligned} \tag{C5}$$

Again, with the FOC $w.r.t$ α

$$J_W W(\hat{g}_{P,t} + l^{-1} - r) + J_{WW}\alpha_t^2 W^2 \sigma_{P,t}^2 = 0$$

and substitute $J_W = \rho/W_t$ and $J_{WW} = -\rho\frac{1}{W^2}$, I get

$$\alpha_t = \frac{\hat{g}_{P,t} + l^{-1} - r}{\sigma_{P,t}^2}, \tag{C6}$$

The fundamental investors' linear demand function, the geometric Brownian form of the dividend process and the logarithmic form of extrapolators' utility function together indicate a linear relation between the equilibrium price P_t and the dividend process D_t. For analytical convenience, I denote W_t as the aggregate wealth and define $x_t \equiv \frac{W_t}{D_t}$ and $l(S_t, x_t) \equiv \frac{P_t}{D_t}$, which follows

$$\begin{aligned} dx_t =& g_{x,t}(S_t, x_t)dt + \sigma_{x,t}(S_t, x_t)d\omega_t \\ =& \hat{g}_{x,t}(S_t, x_t)dt + \sigma_{x,t}(S_t, x_t)d\omega_t^e. \end{aligned} \tag{C7}$$

With extrapolators' optimal strategy, I can solve for the dynamics of x_t as follows

$$dx_t = x_t(r - \rho + \alpha_t[\hat{g}_{P,t} + l^{-1} - r] - \hat{g}_{D,t} + \sigma_D^2 - \alpha_t\sigma_D\sigma_{P,t})dt + x_t(\alpha_t\sigma_{P,t} - \sigma_D)d\omega_t^e, \tag{C8}$$

which yields

$$\begin{aligned} \hat{g}_{x,t}(S_t, x_t) =& x_t(r - \rho + \alpha_t[\hat{g}_{P,t} + l^{-1} - r] - \hat{g}_{D,t} + \sigma_D^2 - \alpha_t\sigma_D\sigma_{P,t}) \\ \sigma_{x,t}(S_t, x_t) =& x_t(\alpha_t\sigma_{P,t} - \sigma_D). \end{aligned} \tag{C9}$$

To solve for $\hat{g}_{P,t}$ and $\hat{g}_{D,t}$, I consider $l(S_t, x_t) \equiv \frac{P_t}{D_t}$ and apply Ito's lemma on both sides of it:

$$\begin{aligned} RHS = dl =& l_x dx + l_S dS + 1/2 l_{xx}(dx)^2 + 1/2 l_{SS}(dS)^2 + l_{Sx}(dx)(dS) \\ =& (l_x\hat{g}_x + l_S\mu_S + 1/2 l_{xx}\sigma_x^2 + 1/2 l_{SS}\sigma_S^2 + l_{Sx}\sigma_x\sigma_S)dt + \\ & + (l_x\sigma_x + l_S\sigma_S)\omega_t^e, \end{aligned} \tag{C10}$$

and

$$LHS = d(\frac{P}{D}) = \frac{P}{D}[\frac{dP}{P} - \frac{dD}{D} + (\frac{dP}{P})^2 - (\frac{dP}{P})(\frac{dD}{D})] \tag{C11}$$
$$= l\Big((\hat{g}_{P,t} - \hat{g}_{D,t} + \sigma_D^2 - \sigma_D \sigma_{P.t})dt +$$
$$+ (\sigma_{P,t} - \sigma_D)\omega_t^e\Big).$$

By matching terms, I get

$$\sigma_{P,t} = \sigma_D + l^{-1}(l_x \sigma_x + l_S \sigma_S), \tag{C12}$$

and

$$\hat{g}_{P,t} = \hat{g}_{D,t} - \sigma_D^2 + \sigma_D \sigma_{P.t} + l^{-1}(l_x \hat{g}_x + l_S \mu_S + 1/2 l_{xx} \sigma_x^2 + 1/2 l_{SS} \sigma_S^2 + l_{Sx} \sigma_x \sigma_S) \tag{C13}$$

Also, by the market clearing condition (20) in the main text, I have

$$\mu \alpha_t W_t + (1-\mu)Q = P_t \tag{C14}$$
$$\mu \alpha_t x_t + \frac{1-\mu}{k(r-g_D)} = \frac{1-\mu+k}{k} l$$
$$\alpha_t x_t = \frac{1-\mu+k}{k\mu} l - \frac{1-\mu}{\mu k(r-g_D)} \equiv c_0 l - c_1$$

Then I solve for $\sigma_{P,t}$. substitute (C9) into (C12), and use the market clearing condition, I can get

$$\sigma_x = \alpha_t x \sigma_{P,t} - \sigma_D x = (c_0 l - c_1)\sigma_{P,t} - x\sigma_D \tag{C15}$$

Substitute the above equation and equation (10) into equation (C12), I get a function for σ_P:

$$(l_x/l(c_0 l - c_1) - 1)\sigma_{P,t}^2 - (l_x/lx - 1)\sigma_D \sigma_{P,t} + l_s/l\theta(\mu_H - S)(S - \mu_L) = 0, \tag{C16}$$

and we get the expression for σ_P

$$\sigma_{P,t} = \frac{(\frac{l_x}{l}x-1)\sigma_D - \sqrt{(\frac{l_x}{l}x-1)^2\sigma_D^2 - 4(\frac{l_x}{l}(c_0 l - c_1)-1)\frac{l_S}{l}\theta(\mu_H - S)(S-\mu_L)}}{2(\frac{l_x}{l}(c_0 l - c_1)-1)}, \tag{C17}$$

There are two roots to the equation (C16) and by taking $x \to 0$, I can easily exclude one of them.

The PDE for function $l(S,x)$ can be obtained by combining the market clearing condition and extrapolators' optimal trading strategy:

$$\alpha_t = \frac{c_0 l - c_1}{x} = \frac{\hat{g}_{P,t} + l^{-1} - r}{\sigma_{P,t}^2}. \tag{C18}$$

Note that σ_P is a nonlinear function of l, l_x and l_S. To solve it, I apply Chebyshev projection method described in the appendix.

Then further we can get expressions for g_x. First, by combining the drift terms in equation (C11) and (C10), I have

$$l(\hat{g}_{P,t} - \hat{g}_{D,t} + \sigma_D^2 - \sigma_D\sigma_{P,t}) = l_x\hat{g}_x + l_S\mu_S + 1/2 l_{xx}\sigma_x^2 + 1/2 l_{SS}\sigma_S^2 + l_{Sx}\sigma_x\sigma_S, \tag{C19}$$

and with the definition of $\hat{g}_x$ in equation (C9), I get

$$\begin{aligned}\hat{g}_{D,t} =& \frac{xl_x/l(r-\rho+\alpha_t(\hat{g}_{P,t}+l^{-1}-r)+\sigma_D^2-\sigma_D\sigma_{P,t}\alpha_t) + l_s/l\mu_S + 1/2\frac{l_{xx}}{l}\sigma_x^2}{xl_x/l-1} \\ &+ \frac{1/2\frac{l_{ss}}{l}\sigma_s^2 + \frac{l_{sx}}{l}\sigma_S\sigma_x - (1-\theta)g_D - \theta S - \sigma_D^2 + \sigma_D\sigma_{P,t}}{xl_x/l-1},\end{aligned} \tag{C20}$$

I can get the expression for the risky asset growth rate under the true probability measure:

$$g_{P,t} = \hat{g}_{P,t} + \sigma_{P,t}/\sigma_D(g_D - \hat{g}_{D,t}) \tag{C21}$$

Then $\hat{g}_x$ becomes

$$\hat{g}_x = x(r-\rho+\alpha_t(\hat{g}_{P,t}+l^{-1}-r) - \hat{g}_{D,t} + \sigma_D^2 - \alpha_t\sigma_D\sigma_{P,t}) \tag{C22}$$

For the boundary conditions, when $x_t \to 0$, fundamental investors dominate and risky asset price is mainly driven by fundamental investors' demand; by the market clearing condition (20), I have

$$\lim_{x_t \to 0} l = \frac{c_1}{c_0}, \tag{C23}$$

where $c_0 = \frac{k+1-\mu}{k\mu}$ and $c_1 = \frac{1-\mu}{k\mu(r-g_D)}$ are both constant. Furthermore,

$$\begin{aligned}\lim_{x \to 0} l_x &= \lim_{x \to 0} \frac{l(S,x) - l(S,0)}{x} \\ &= \lim_{x \to 0} \frac{\alpha x / c_0 + \frac{c_1}{c_0} - \frac{c_1}{c_0}}{x} \\ &= \lim_{x \to 0} \frac{\hat{g}_{P,t} + \frac{c_0}{c_1} - r}{\sigma_D^2} \frac{1}{c_0} \equiv \frac{\alpha(S,0)}{c_0}.\end{aligned} \tag{C24}$$

For the case where $x_t \to \infty$, extrapolators dominate and in order to clear the market, α goes to 0; otherwise the asset prices goes to infinity. This means that the conditional excess returns perceived by extrapolators are zero, leading to

$$\lim_{x \to \infty} l = (r - \hat{g}_{P,t})^{-1}. \tag{C25}$$

Chebyshev polynomial requires a domain of $[-1, 1]$. Therefore, the following transformations maps S and x to new variables that lie between -1 and 1:

$$\begin{aligned} y =& aS + b, \\ a =& \frac{2}{\mu_H - \mu_L}, \quad b = -\frac{\mu_H + \mu_L}{\mu_H - \mu_L}, \\ z =& \frac{x - \zeta}{x + \zeta}, \end{aligned} \tag{C26}$$

where ζ is some constant. When $S = \mu_L$,

$$y = \frac{2\mu_L}{\mu_H - \mu_L} - \frac{\mu_H + \mu_L}{\mu_H - \mu_L} = -1; \tag{C27}$$

when $S = \mu_H$,

$$y = \frac{2\mu_H}{\mu_H - \mu_L} - \frac{\mu_H + \mu_L}{\mu_H - \mu_L} = 1; \tag{C28}$$

Similarly, when $x = 0$, $z = -1$ and when $x = +\infty$, $z = 1$

The following equalities prove to be useful:

$$x = \frac{\zeta z + \zeta}{1 - z}, \tag{C29}$$

and

$$\frac{\partial z}{\partial x} = \frac{(1-z)^2}{2\zeta} \tag{C30}$$

Further, I define $l(S,x) = l(\frac{y-b}{a}, \frac{\zeta(z+1)}{1-z}) \equiv m(y,z)$. Then the corresponding derivatives of $m(y,z)$ follow

$$\begin{aligned} l_x =& \frac{\partial z}{\partial x} m_z = \frac{(1-z)^2}{2\zeta} m_z, \\ l_S =& \frac{\partial y}{\partial S} m_y = a m_y, \\ l_{xx} =& \frac{(1-z)^3}{(2\zeta)^2}[(1-z)m_{zz} - 2m_z] \\ l_{SS} =& a^2 m_{yy}, \\ l_{Sx} =& a\frac{(1-z)^2}{2\zeta} m_{yz}, \end{aligned} \tag{C31}$$

Substitute all the terms into the PDE and its boundary conditions, I get

$$\lim_{z \to -1} m(y,z) = \frac{c_1}{c_0} \tag{C32}$$

$$\lim_{z \to -1} m_z(y,z) = \frac{\zeta}{2}\frac{1}{c_0}\alpha(S,0) \equiv \frac{\zeta}{2c_0}\alpha_0 \tag{C33}$$

$$\lim_{z \to 1} m(y,z) = (r - \hat{g}_{P,t})^{-1} = \frac{1}{r - (1-\theta)g_D - \frac{\theta}{a}(y-b)} \tag{C34}$$

where $\alpha_0 = \frac{(1-\theta)g_D + \frac{\theta}{a}(y-b) + \frac{c_0}{c_1} - r}{\sigma_D^2}$ and $\alpha_{0,y} = \frac{\theta}{a\sigma_D^2}$. I use Chebyshev polynomial to approximate it.

To be specific, I set $m(y, z)$ as follows:

$$m(y,z) \equiv \frac{c_1}{c_0} + \frac{\zeta}{2c_0}(1+z)\alpha_0 + (1+z)^2 \sum_{i+j \leq N} a_{ij} T_i(z) T_j(y) \tag{C35}$$

$$\equiv \frac{c_1}{c_0} + \frac{\zeta}{2c_0}(1+z)\alpha_0 + (1+z)^2 v(y,z), \tag{C36}$$

where $v(y, z) = \sum_{i+j \leq N} a_{ij} T_i(z) T_j(y)$. Here $T_i(z)$ and $T_j(y)$ are Chebyshev Polynomial functions and $a_{i,j}$ are the coefficients to be determined. Moreover,

$$m_z(y,z) = \frac{\zeta}{2c_0}\alpha_0 + 2(1+z)v(y,z) + (1+z)^2 v_z(y,z) \tag{C37}$$

$$m_{zz}(y,z) = 4(1+z)v_z(y,z) + 2v(y,z) + (1+z)^2 v_{zz}(y,z) \tag{C38}$$

$$m_y(y,z) = \frac{\zeta}{2c_0}(1+z)\alpha_{0,y} + (1+z)^2 v_y(y,z) \tag{C39}$$

$$m_{yy}(y,z) = (1+z)^2 v_{yy}(y,z) (\text{since } w_{yy} = 0) \tag{C40}$$

$$m_{zy}(y,z) = \frac{\zeta}{2c_0}\alpha_{0,y} + 2(1+z)v_y(y,z) + (1+z)^2 v_{zy}(y,z) \tag{C41}$$

$$\tag{C42}$$

and boundary conditions (C23) and (C24) hold automatically. Here $T_i(z)$ and $T_j(y)$ are Chebyshev polynomials evaluated at z and y respectively. Therefore, I need to minimize

$$\sum_{ij}^{M} \beta_{ij}^1 \Big[\frac{c_0 m(x_i, y_j) - c_1}{x_i} - \frac{\hat{g}_P(x_i, y_j) + m(x_i, y_j)^{-1} - r}{\sigma_P^2(x_i, y_j)}\Big]^2 \tag{C43}$$
$$+K \sum_j \beta_j^2 \Big[m(y_j, 1) - (r - \hat{g}_{P,t})^{-1}\Big]^2$$

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Figure 1. Investor Expectations in Gallup and S&P 500 Index

In the sample period 1996:10 - 2011:11, I plot both investor expectation index in Gallup survey and the S&P500 index. The blue solid line plots Gallup Index, defined as the percentage difference between bullish investors and bearish investors. The red dash line plot the S&P500 index. The shaded areas represent NBER recessions.

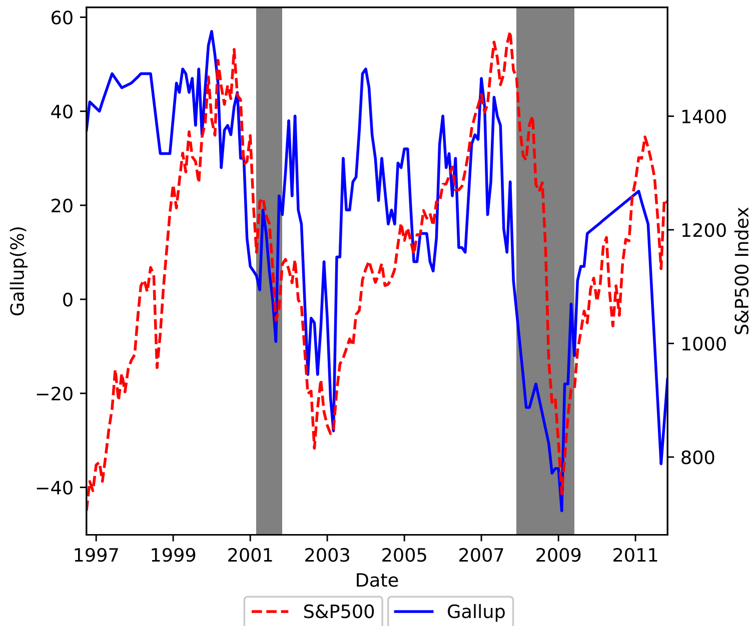


Figure 2. Investor Expectations in Gallup and Household Mutual Fund Flows

In the sample period 1996:10 - 2011:11, I plot both investor expectation index in Gallup survey and the household flows from HNPO sector. The blue solid line plots Gallup Index and red dashed line plots the household mutual fund flows. The shaded areas represent NBER recessions.

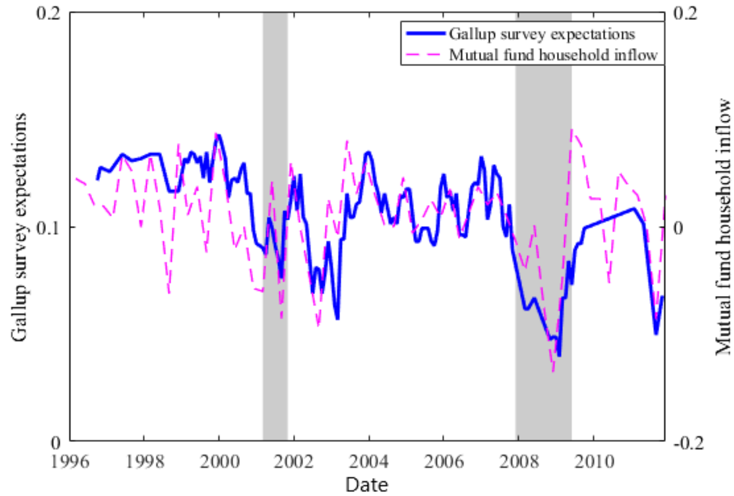


Figure 3. Predictive power of investor sentiment when the wealth level is low

This figure provides intuition for the positive predictive pattern of investor sentiment when the wealth is low. When the wealth level is low, extrapolators have low market impact and they are very unlikely to push asset prices away from the fundamental values. In this situation, investor sentiment reflects market correction and therefore positively predict future returns.

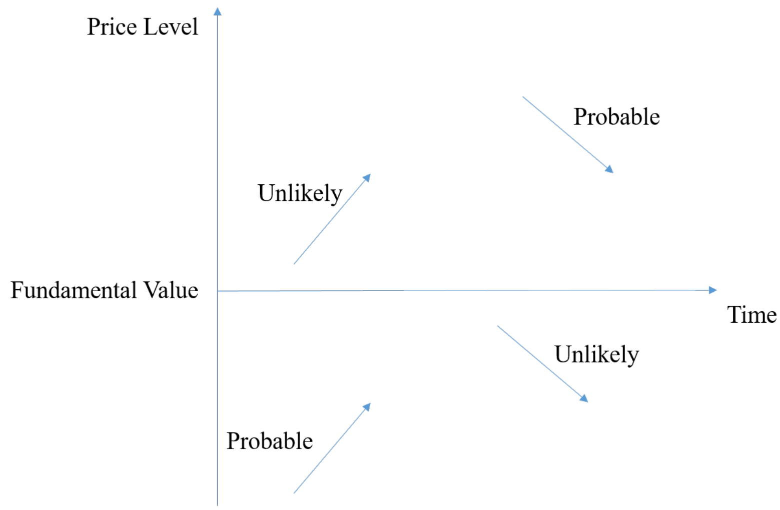


Figure 4. Calibrated Model Solutions: Rational Benchmark.

This figure plots the price to dividend ratio l, the optimal portfolio α_t and the return volatility σ_P as functions of both the latent state variable S_t and the transformed wealth to dividend ratio z_t in the rational benchmark model. As z_t goes to -1 (1), the wealth to dividend ratio $\frac{W_t}{D_t}$ goes to zero (infinity). The parameter values are: $r = 4\%$, $g_D = 1.5\%$, $\rho = 2\%$, $k = 0.5$, $\mu = 0.5$, $\mu_H = 3\%$, $\mu_L = -9\%$, $\chi = 0.15$, $\lambda = 0.15$.

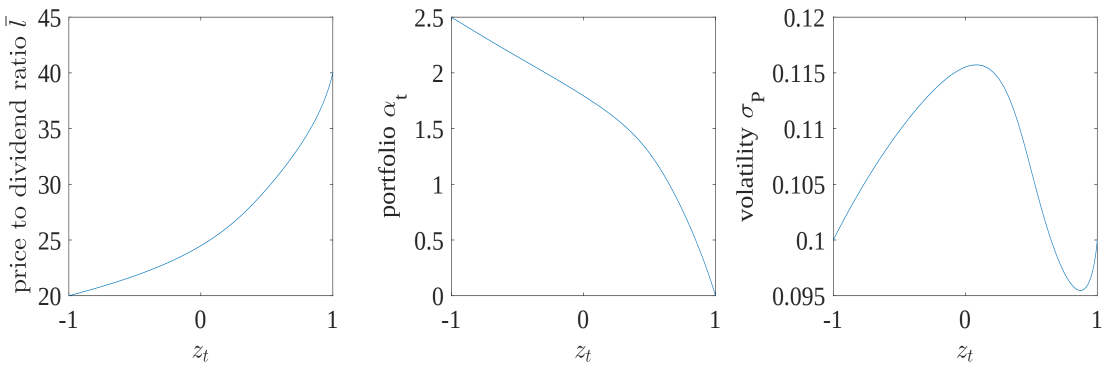


Figure 5. Calibrated Model Solutions: Behavioral Model.

This figure plots the price to dividend ratio l, the optimal portfolio α_t and the return volatility σ_P as functions of both the latent state variable S_t and the transformed wealth to dividend ratio z_t in the behavioral model. As z_t goes to -1 (1), the wealth to dividend ratio $\frac{W_t}{D_t}$ goes to zero (infinity). The parameter values are: $r = 4\%$, $g_D = 1.5\%$, $\rho = 2\%$, $k = 0.5$, $\mu = 0.5$, $\mu_H = 3\%$, $\mu_L = -9\%$, $\chi = 0.15$, $\lambda = 0.15$.

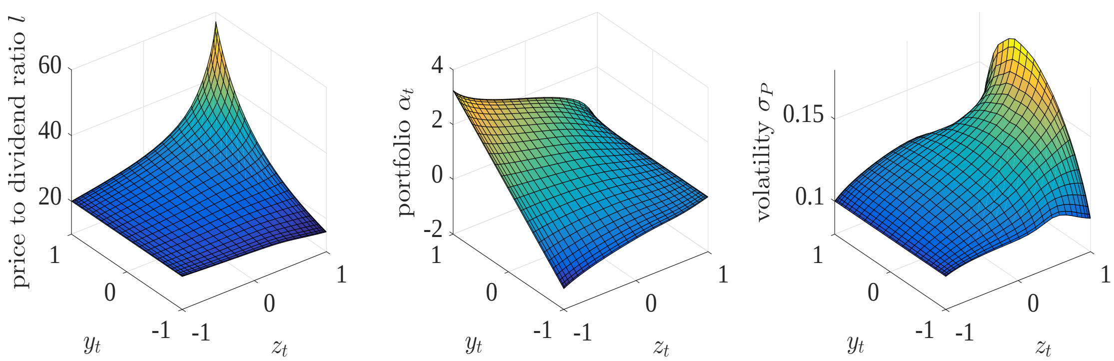


Figure 6. Gallup Survey and Simulated Investor Sentiment.

In the sample period 1996:10 - 2011:11, I plot both investor expectation index in Gallup survey and investor sentiment based on model simulations. The blue solid line plots Gallup Index and red dashed line plots the investor sentiment based on model simulations. The shaded areas represent NBER recessions. The parameter values are: $r = 4\%$, $g_D = 1.5\%$, $\mu_H = 0.03\%$, $\mu_L = -0.06\%$, $\chi = \lambda = 10\%$, $\theta = 0.5\%$, $\rho = 2\%$, $k = 0.5$, $\mu = 0.5$, $\mu_H = 3\%$, $\mu_L = -3\%$.

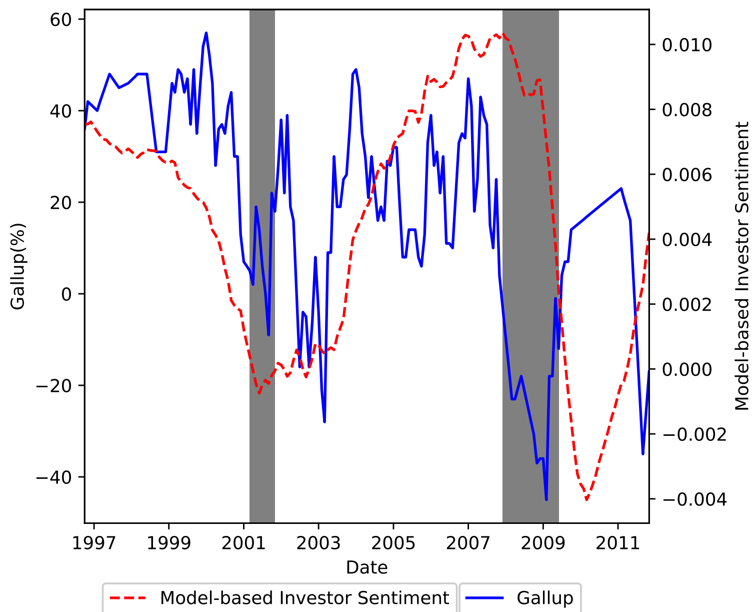
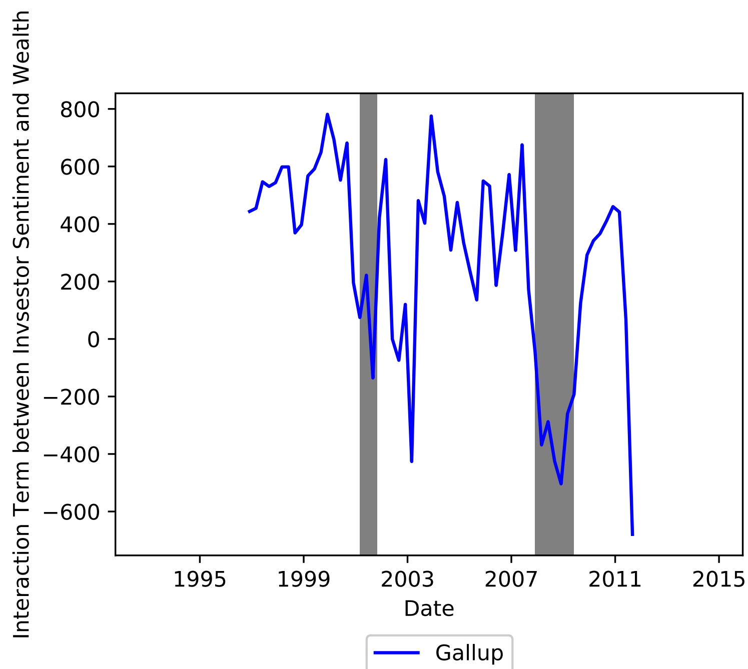


Figure 7. Interaction between wealth and investor sentiment vs degree of extrapolation.

This figure plots Interaction between wealth and investor sentiment (upper panel) vs degree of extrapolation (lower panel). The lower panel is based on Cassella and Gulen (2015). The parameter values are: $r = 4\%$, $g_D = 1.5\%$, $\mu_H = 0.03\%$, $\mu_L = -0.06\%$, $\chi = \lambda = 10\%$, $\theta = 0.5\%$, $\rho = 2\%$, $k = 0.5$, $\mu = 0.5$, $\mu_H = 3\%$, $\mu_L = -3\%$.



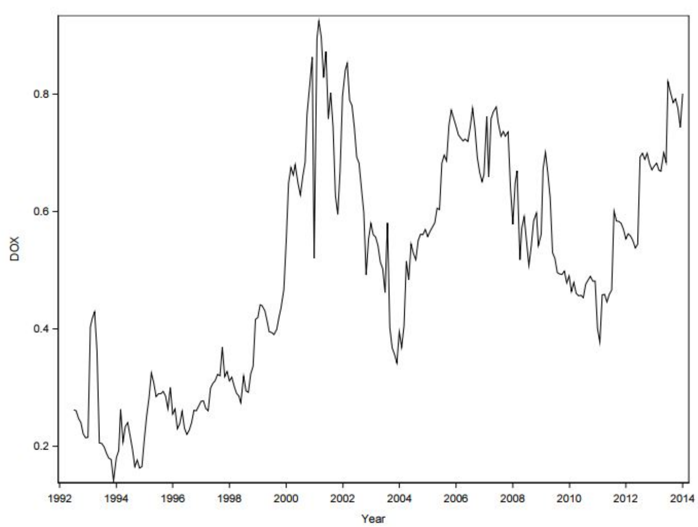


Figure 8. Perceived Sharpe Ratio and True Sharpe Ratio.

This figure plots the true Sharpe ratio (red dash line) and perceived Sharpe Ratio (blue solid line) based on model simulation. The parameter values are: $r = 4\%$, $g_D = 1.5\%$, $\mu_H = 0.03\%$, $\mu_L = -0.06\%$, $\chi = \lambda = 10\%$, $\theta = 0.5\%$, $\rho = 2\%$, $k = 0.5$, $\mu = 0.5$, $\mu_H = 3\%$, $\mu_L = -3\%$.

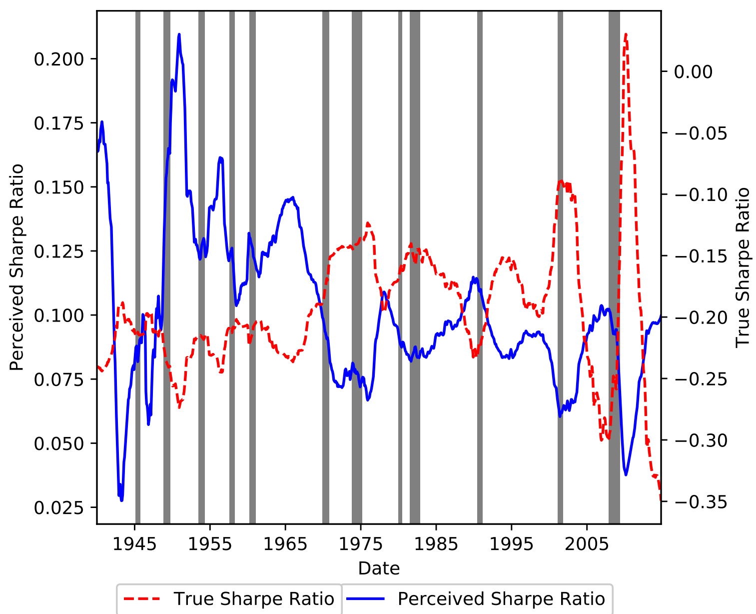


Table I. Summary statistics

This table reports the summary statistics and correlation matrix between key variables. Panel A reports the general information for the Gallup survey measurement of investor sentiment, excess returns over next twelve months, wealth to dividend ratio for the HNPO sector, and the Psentiment variable. Panel B reports the correlation matrix for these key variables.

Statistic	N	Mean	St. Dev.	Min	Max
Gallup Measurement of Investor Sentiment	182	0.107	0.011	0.077	0.124
Excess Return (next 12-months)	912	0.086	0.174	−0.471	0.611
Wealth to dividend ratio	183	0.984	0.599	0.137	2.137
Psentiment	492	0.023	0.011	−0.024	0.047

Panel A: General information

	Gallup_rescaled	CRSPex12	WD	Psentiment
Gallup Measurement of Investor Sentiment	1.00	-0.02	-0.32	0.75
Excess Return (next 12-months)	-0.02	1.00	0.03	-0.12
Wealth to dividend ratio	-0.32	0.03	1.00	-0.28
Psentiment	0.75	-0.12	-0.28	1.00

Panel B: Correlation matrix for the key variables

Table II. Forecast Revision of Investor Sentiment

This table reports the results from the following predictive regression of N-months ahead sentiment revision based on current sentiment level:

$$SR_{t+1}[\text{Sent}_{t+N} - \text{Sent}_{t+1}] = c + d\text{Sent}_t + u_t, \tag{44}$$

where on the left hand side $SR_{t+1}[\text{Sent}_{t+N} - \text{Sent}_{t+1}]$ measures the changes in investor sentiment over future N horizons, Sent_t represents the investor sentiment at time t and u_t on the right hand side is the corresponding residual at time t. Panel A reports the regression results based on the Gallup survey. Panel B reports the regression results based on the Psentiment. All standard errors in parenthesis in parentheses are again based on the Newey-West correction (Newey and West (1986)).

Panel A: Forecast Revision of Investor Sentiment: Gallup

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	0.015**	0.033***	0.066***	0.081***
	(0.006)	(0.011)	(0.015)	(0.015)
Gallup	−0.142***	−0.313***	−0.624***	−0.774***
	(0.053)	(0.099)	(0.142)	(0.149)
Observations	179	176	170	164
R^2	0.062	0.132	0.284	0.359
Adjusted R^2	0.057	0.127	0.279	0.355

Panel B: Forecast Revision of Investor Sentiment: Psentiment

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	0.001**	0.004**	0.008***	0.011***
	(0.001)	(0.001)	(0.002)	(0.002)
Psentiment	−0.064**	−0.158***	−0.329***	−0.477***
	(0.026)	(0.053)	(0.084)	(0.086)
Observations	489	486	480	474
R^2	0.031	0.080	0.162	0.235
Adjusted R^2	0.029	0.078	0.160	0.233

Note: *p<0.1; **p<0.05; ***p<0.01

Table III. Predictive regressions on Future Returns

This table reports the results from the following predictive regression of N-months ahead returns from CRSP value-weighted index:

$$R^e_{t+N} = a_0 + a_1 \text{Sent}_t + \epsilon_t \tag{45}$$

where R^e_{t+N} represents the excess return over the next N-month, Sent_t represents quantitative investor sentiment variable measured by Gallup survey. Results for future 1 to 6 quarters are reported in the column 1 to 4. Panel A reports the predictive regression results based on the investor sentiment in Gallup survey. All standard errors in parentheses are adjusted using Newey-West correction. The sample spans from 1996:10 to 2011:11.

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
	(1)	(2)	(3)	(4)
Constant	−0.079	−0.054	0.090	0.372
	(0.134)	(0.255)	(0.409)	(0.450)
Gallup	0.850	0.730	−0.400	−2.819
	(1.201)	(2.292)	(3.711)	(4.230)
Observations	182	182	182	182
R^2	0.011	0.004	0.0005	0.015
Adjusted R^2	0.005	-0.002	-0.005	0.010
Note:				*p<0.1; **p<0.05; ***p<0.01

Table IV. Conditional Predictive regressions on Future Returns

This table reports the results from the following predictive regression of N-months ahead returns from CRSP value-weighted index:

$$R^e_{t+N} = a + b\text{Sent}_t + cW_t/D_t + d\text{Sent}_t \times W_t/D_t + \epsilon_t \tag{46}$$

where R^e_{t+N} represents the excess return over the next N-month, Sent_t represent investor sentiment variable, W_t represent the total financial asset value of the HNPO sector. Results for future 1 to 6 quarters are reported in the column 1 to 4. Panel A reports the predictive regression results based on the investor sentiment in Gallup survey. Panel B reports the predictive regression results based on Psentiment. All standard errors in parentheses are adjusted using Newey-West correction. The sample spans from 1996:10 to 2011:11.

Panel A: Predictive regression based on Gallup

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	11.200***	23.683***	29.549**	50.552**
	(3.787)	(8.977)	(14.805)	(20.127)
Gallup	0.741***	1.638**	2.081**	3.729***
	(0.265)	(0.716)	(0.996)	(1.410)
W_t/D_t	−7.180***	−15.916**	−20.573**	−35.970***
	(2.474)	(6.738)	(8.892)	(13.543)
Gallup × W_t/D_t	−1.150***	−2.420**	−2.942*	−5.183**
	(0.430)	(0.973)	(1.685)	(2.125)
Observations	60	60	60	60
R^2	0.042	0.087	0.088	0.143
Adjusted R^2	-0.009	0.038	0.039	0.097

Note: *p<0.1; **p<0.05; ***p<0.01

Panel B: Predictive regression based on Psentiment

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	20.384***	37.181***	54.921***	84.599***
	(3.067)	(5.329)	(16.467)	(21.284)
Psentiment	0.023***	0.041***	0.059***	0.098***
	(0.005)	(0.010)	(0.021)	(0.027)
W_t/D_t	−1.496***	−2.776***	−4.161***	−6.398***
	(0.208)	(0.350)	(1.159)	(1.546)
Psentiment×	−0.323***	−0.546***	−0.753**	−1.291***
W_t/D_t	(0.090)	(0.162)	(0.340)	(0.421)
Observations	60	60	60	60
R^2	0.179	0.307	0.367	0.524
Adjusted R^2	0.135	0.270	0.333	0.499

Note: *p<0.1; **p<0.05; ***p<0.01

Table V. Conditional Predictive regressions on Future Returns

This table reports the results from the following predictive regression of N-months ahead returns from CRSP value-weighted index:

$$R^e_{t+N} = a + b\text{Sent}_t + cW_t/D_t + d\text{Sent}_t \times W_t/D_t + X_t + \epsilon_t \tag{47}$$

where R^e_{t+N} represents the excess return over the next N-month, Sent_t represent Gallup survey measurement of investor sentiment, W_t represent the total financial asset value of the HNPO sector. Results for future 1 to 6 quarters are reported in the column 1 to 4. X_t represent the commonly used forecasters for the aggregate future market returns, cay. All standard errors in parentheses are adjusted using Newey-West correction. The sample spans from 1996:10 to 2011:11.

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	15.277**	30.900***	40.223***	66.621***
	(6.108)	(9.780)	(14.455)	(10.360)
Gallup	0.077**	0.164**	0.195*	0.363***
	(0.033)	(0.070)	(0.113)	(0.064)
W_t/D_t	−0.799**	−1.673**	−2.008*	−3.633***
	(0.319)	(0.678)	(1.081)	(0.623)
Gallup × W_t/D_t	−0.007***	−0.013***	−0.023***	−0.033***
	(0.002)	(0.004)	(0.005)	(0.004)
cay	0.194	1.191	4.062*	4.613
	(0.604)	(1.033)	(2.398)	(3.102)
Constant	−1.312**	−2.694***	−3.314**	−5.764***
	(0.596)	(0.987)	(1.543)	(1.041)
Observations	60	60	60	60
R^2	0.162	0.317	0.493	0.625
Adjusted R^2	0.085	0.254	0.446	0.590
Note:				*p<0.1; **p<0.05; ***p<0.01

Table VI. Parameter Values

Parameter	Variable	Value
Asset parameter:		
Expected dividend growth	g_D	1.5%
Standard deviation of dividend growth	σ_D	10%
Correlation between dD and dC	ρ	0.2
Risk-free rate	r	4%
Utility parameter:		
Subjective discount factor	δ	0.02
Belief parameter:		
Degree of extrapolation	θ	0.5
Transition intensity from H to L	χ	0.10
Transition intensity from L to H	λ	0.10
Return in state H	μ_H	0.03
Return in state L	μ_L	-0.06
Other parameter:		
Demand parameter for fundamental investors	k	0.5
Population fraction	μ	0.5

Table VII. Simulated Extrapolative Expectations: Behavioral Models

This table reports the results from the following regression focusing on the determinants of investor sentiment, based on the simulations of the behavioral model:

$$\mathbb{E}^e[R_{t+12}]_t = a_0 + a_1 X_t + \epsilon_t \tag{48}$$

where $\mathbb{E}[R_{t+12}]_t$ represents the perceived expected returns at time t, X_t represent either past accumulative 12-month returns (R12) or the current log price to dividend ratio (logPD). Results in column 1 to 2 are based on the perceived expectations of returns with dividend yield. Results in column 1 to 2 are based on the perceived expectations of returns without dividend yield. All standard errors in parentheses are corrected based on Newey-west approach.

	Dependent variable:			
	Expectations with dividend yield		Expectations without dividend yield	
	(1)	(2)	(3)	(4)
R12	0.023**		0.011**	
	(0.011)		(0.006)	
logPD		0.100***		0.051***
		(0.005)		(0.003)
Constant	−0.050***	−0.328***	−0.016***	−0.160***
	(0.012)	(0.014)	(0.006)	(0.007)
Observations	239	239	239	239
R^2	0.235	0.914	0.221	0.907
Adjusted R^2	0.232	0.914	0.218	0.907

Note: *p<0.1; **p<0.05; ***p<0.01

Table VIII. Memory Structure of Investor Expectations: Behavioral Models

The table reports the memory decay parameter ψ, the intercept a, the regression coefficient b, and the adjusted R-squared, for running the non-linear least squares regression

$$\text{Expectation}_t = a + b \sum_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t,$$

over a sample of 15 years or 50 years, where $w_j = e^{-\psi(j-1)\Delta t}/\Sigma_{l=1}^{n} e^{-\psi(l-1)\Delta t}$. Here $\Delta t = 1/12$ and $n = 600$.

	$\mathbb{E}^e_t[R^D_{t+dt}]$	
ϕ	0.510	0.521
a	0.064	0.064
b	1.15	1.18
R^2	0.78	0.78

Table IX. Forecast Revision of Investor Sentiment: Simulated Results Based on Behavioral Model

This table reports the results from the following predictive regression of N-months ahead sentiment revision based on current sentiment level, based on the simulated series from the behavioral model:

$$SR_{t+1}[\text{Sent}_{t+N} - \text{Sent}_{t+1}] = c + d\text{Sent}_t + u_t, \tag{49}$$

where on the left hand side $SR_{t+1}[\text{Sent}_{t+N} - \text{Sent}_{t+1}]$ measures the changes in investor sentiment over future N horizons, Sent_t represents the investor sentiment at time t and u_t on the right hand side is the corresponding residual at time t. Panel A reports the regression results based on the Gallup survey. Panel B reports the regression results based on the Psentiment. All standard errors in parentheses are again based on the Newey-West correction.

Panel A: Forecast Revision of Investor Sentiment: with Dividend Yield

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	0.001	0.005*	0.018***	0.036***
	(0.001)	(0.003)	(0.006)	(0.009)
Sent	−0.021	−0.090*	−0.332***	−0.666***
	(0.022)	(0.051)	(0.108)	(0.164)
Observations	236	233	227	221
R^2	0.015	0.052	0.173	0.335
Adjusted R^2	0.011	0.048	0.170	0.332

Panel B: Forecast Revision of Investor Sentiment: without Dividend Yield

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
Constant	0.0001	0.0004**	0.001***	0.002***
	(0.0001)	(0.0002)	(0.0004)	(0.001)
Sent	−0.022	−0.080**	−0.253***	−0.468***
	(0.015)	(0.034)	(0.061)	(0.083)
Observations	896	893	887	881
R^2	0.017	0.048	0.137	0.242
Adjusted R^2	0.016	0.047	0.136	0.242
Note:				*p<0.1; **p<0.05; ***p<0.01

Table X. Model-simulated Conditional Predictive regressions on Future Returns: Behavioral Models

This table reports the results from the following predictive regression of N-months ahead returns based on model simulations:

$$R^e_{t+N} = a + b\mathrm{Sent}_t + cW_t/D_t + d\mathrm{Sent}_t \times W_t/D_t + \epsilon_t \tag{50}$$

where R^e_{t+N} represents the excess return over the next N-month, Sent_t represent investor sentiment variable, W_t/D_t represent the wealth to dividend ratio in the model. Results for future 1 to 6 quarters are reported in the column 1 to 4. All standard errors in parentheses are adjusted using Newey-West correction.

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
	(1)	(2)	(3)	(4)
Constant	0.979***	0.955***	0.908***	0.870***
	(0.020)	(0.047)	(0.111)	(0.177)
Sent_t	6.714***	13.330**	24.550**	32.641
	(2.201)	(5.184)	(12.465)	(20.182)
W_t/D_t	0.019	0.043	0.101	0.165
	(0.012)	(0.029)	(0.067)	(0.104)
$\mathrm{Sent}_t \times W_t/D_t$	−3.068**	−6.638**	−14.146**	−21.679**
	(1.194)	(2.749)	(6.275)	(9.789)
Observations	239	239	239	239
R^2	0.420	0.496	0.624	0.713
Adjusted R^2	0.413	0.489	0.619	0.709
Note:				*p<0.1; **p<0.05; ***p<0.01

Table XI. Conditional predictive power of investor sentiment.

This table reports the conditional predictive power of investor sentiment S_t, $a_1 + a_3 W_t/D_t$, from the following predictive regression of N-months ahead returns from CRSP value-weighted index:

$$R^e_{t+12} = a + b\text{Sent}_t + cW_t/D_t + d\text{Sent}_t \times W_t/D_t + \epsilon_t \quad (51)$$

where R^e_{t+12} represents the excess return over the next twelve months, Sent_t represent investor sentiment in Gallup survey, W_t represent the total financial asset value of the HNPO sector. The sample spans from 1996:10 to 2011:11.

Investor Sentiment	Wealth Level	Conditional coefficient $b_t = [b + d \times W_t/D_t]$	t-statistics	p-value
Gallup	2	-0.162	-2.660	0.010
	0	0.098	1.428	0.159
	-2	0.359	2.034	0.047
Psentiment	2	-0.267	-3.915	0.000
	0	0.106	2.108	0.039
	-2	0.479	3.020	0.004

Table XII. **Conditional predictive power of investor sentiment: Simulations based on the Behavioral Models.**

This table reports the conditional predictive power of investor sentiment S_t, $a_1 + a_3 W_t/D_t$, from the following predictive regression of N-months ahead returns from CRSP value-weighted index:

$$R^e_{t+12} = a + b\mathrm{Sent}_t + cW_t/D_t + d\mathrm{Sent}_t \times W_t/D_t + \epsilon_t \tag{52}$$

where R^e_{t+12} represents the excess return over the next twelve months, Sent_t represent investor sentiment, W_t/D_t represent the wealth to dividend ratio in the model.

Investor Sentiment	Wealth Level	Conditional coefficient $b_t = [b + d \times W_t/D_t]$	t-statistics	p-value
Behavioral Model	2	-0.025	-3.421	0.000
	0	-0.000	-0.034	0.973
	-2	0.025	2.885	0.004

Table XIII. Leverage ratio and Future Consumption Growth: Behavioral Models

This table reports the predictive power of leverage ratio on future consumption growth rate based on behavioral models.

$$\text{Consumption Growth}_{t+N} = a_0 + a_1\alpha_t + \epsilon_t \tag{53}$$

where α_t represent leverage ratio of extrapolators, and all standard errors in parentheses are adjusted using Newey-West correction.

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
	(1)	(2)	(3)	(4)
α_t	-0.010^{***}	-0.022^{***}	-0.041^{***}	-0.045^{***}
	(0.003)	(0.005)	(0.010)	(0.013)
Constant	0.001^{***}	0.003^{***}	0.006^{***}	0.008^{***}
	(0.0001)	(0.0001)	(0.0002)	(0.0003)
Observations	896	893	887	881
R^2	0.014	0.020	0.021	0.013
Adjusted R^2	0.013	0.019	0.020	0.012
Note:				*p$<$0.1; **p$<$0.05; ***p$<$0.01

Table XIV. Simulated Extrapolative Expectations: Rational Benchmark Models

This table reports the results from the following regression focusing on the determinants of investor sentiment, based on the simulations of the rational benchmark model:

$$\mathbb{E}[R_{t+12}]_t = a_0 + a_1 X_t + \epsilon_t \tag{54}$$

where $\mathbb{E}[R_{t+12}]_t$ represents the perceived expected returns at time t, X_t represent either past accumulative 12-month returns (R12) or the current log price to dividend ratio (logPD). Results in column 1 to 2 are based on the perceived expectations of returns without dividend yield. Results in column 1 to 2 are based on the perceived expectations of returns with dividend yield. All standard errors in parentheses are corrected based on Newey-west approach.

	Dependent variable:			
	Expectations without dividend yield		Expectations with dividend yield	
	(1)	(2)	(3)	(4)
R12	−0.00003		−0.001	
	(0.0002)		(0.002)	
logPD		−0.006***		−0.049***
		(0.001)		(0.0005)
Constant	0.018***	0.037***	0.022***	0.173***
	(0.0003)	(0.002)	(0.002)	(0.001)
Observations	239	239	239	239
R^2	0.0003	0.913	0.016	0.999
Adjusted R^2	-0.004	0.912	0.012	0.999

Note: *p<0.1; **p<0.05; ***p<0.01

Table XV. Model-simulated Conditional Predictive regressions on Future Returns: Rational Benchmark Models

This table reports the results from the following predictive regression of N-months ahead returns based on simulations of the rational benchmark model:

$$R^e_{t+N} = a + b\text{Sent}_t + cW_t/D_t + d\text{Sent}_t \times W_t/D_t + \epsilon_t \tag{55}$$

where R^e_{t+N} represents the excess return over the next N-month, Sent_t represent investor sentiment variable, W_t/D_t represent the wealth to dividend ratio in the model. Results for future 1 to 6 quarters are reported in the column 1 to 4. All standard errors in parentheses are adjusted using Newey-West correction.

	Dependent variable:			
	1 Quarter	2 Quarters	4 Quarters	6 Quarters
	(1)	(2)	(3)	(4)
Constant	1.245***	1.561***	2.307**	3.205**
	(0.163)	(0.392)	(0.954)	(1.630)
Sent_t	−8.969	−20.669	−48.522	−82.221
	(6.303)	(15.138)	(36.854)	(62.965)
W_t/D_t	−0.017**	−0.039**	−0.091**	−0.154*
	(0.008)	(0.019)	(0.046)	(0.079)
$\text{Sent}_t \times W_t/D_t$	0.660**	1.520**	3.596**	6.079**
	(0.307)	(0.726)	(1.744)	(2.994)
Observations	239	239	239	239
R^2	0.047	0.071	0.125	0.176
Adjusted R^2	0.035	0.060	0.114	0.165
Note:			*p<0.1; **p<0.05; ***p<0.01	

Table XVI. Leverage ratio and future consumption growth: rational models

This table reports the predictive power of leverage ratio on future consumption growth rate based on rational models.

$$\text{Consumption Growth}_{t+N} = a_0 + a_1 \alpha_t + \epsilon_t \tag{56}$$

where α_t represent leverage ratio of extrapolators, and all standard errors in parentheses are adjusted using Newey-West correction.

	Dependent variable:			
	WD_growth1 1 Quarter	WD_growth2 2 Quarters	WD_growth3 4 Quarters	WD_growth4 6 Quarters
	(1)	(2)	(3)	(4)
α_t	3.003***	4.500***	6.081***	7.014***
	(0.139)	(0.274)	(0.532)	(0.773)
Constant	0.003***	0.006***	0.012***	0.018***
	(0.0001)	(0.0002)	(0.0004)	(0.001)
Observations	896	893	887	881
R^2	0.341	0.232	0.129	0.086
Adjusted R^2	0.341	0.231	0.128	0.085

Note: *p<0.1; **p<0.05; ***p<0.01